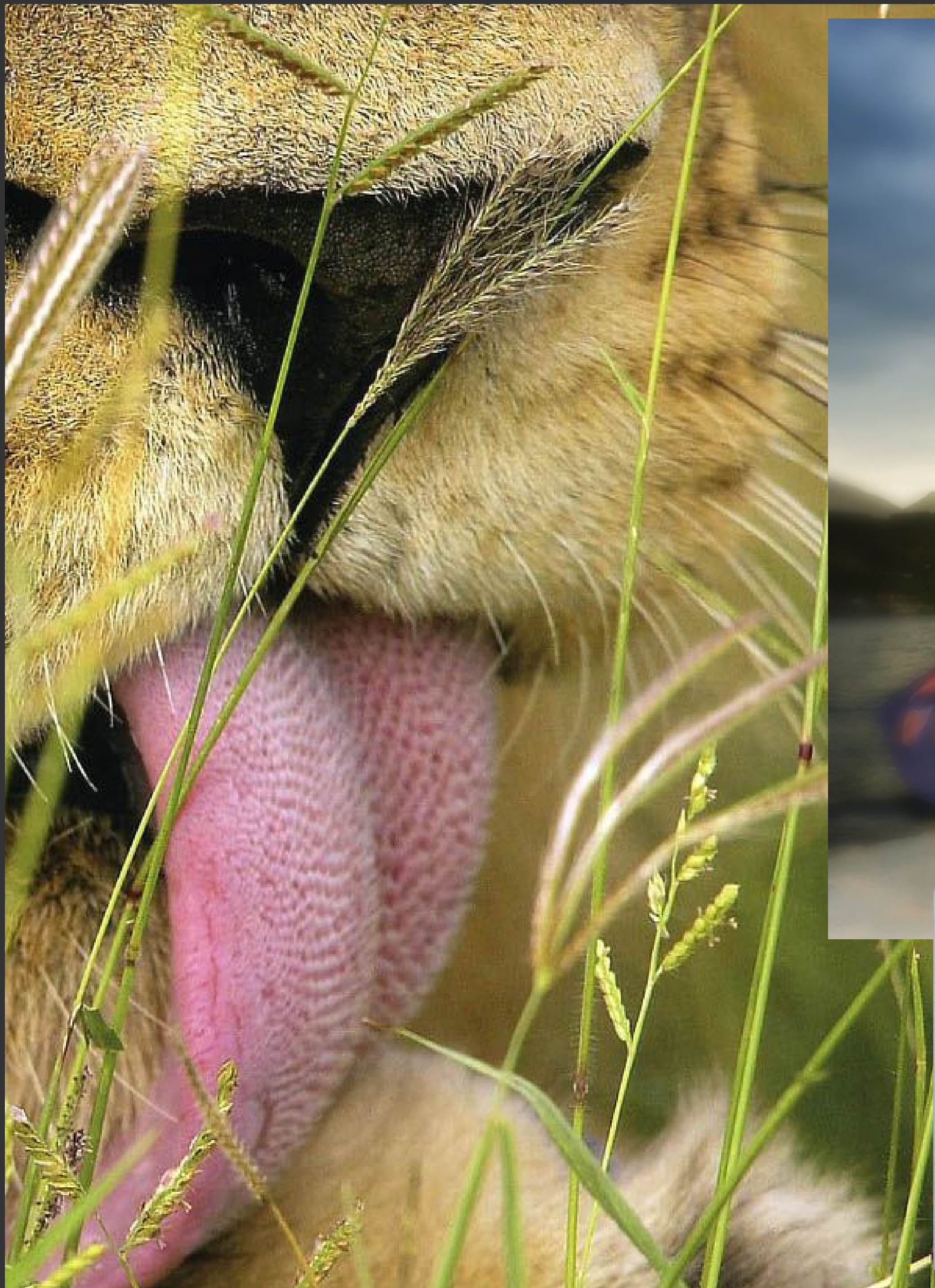
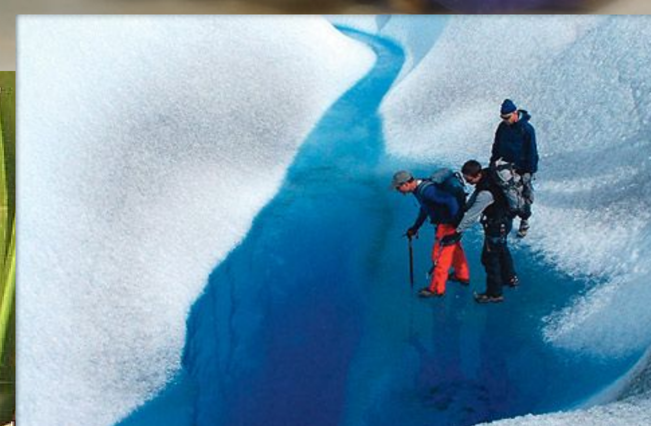
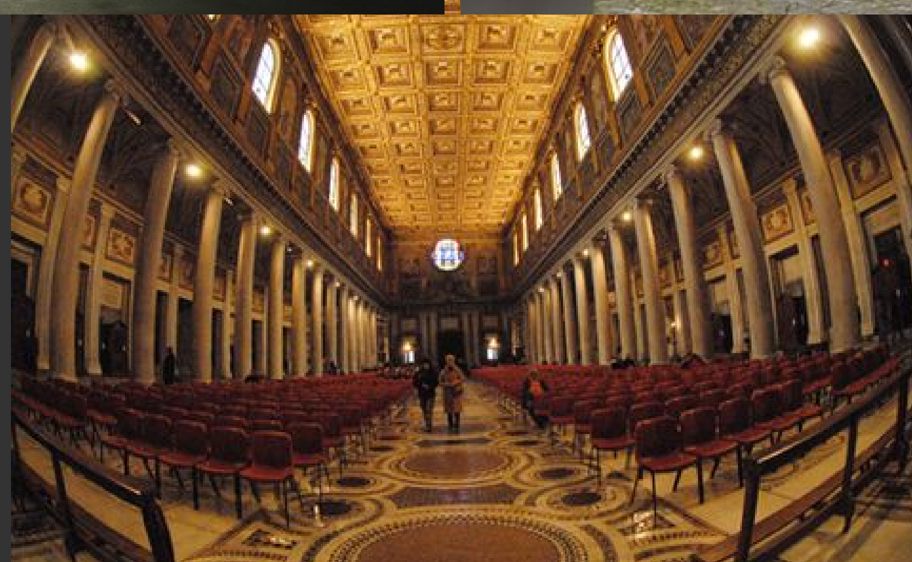
Photographed by Nicéphore Niépce, 1826





Various types of lenses



Adjustable depth of field

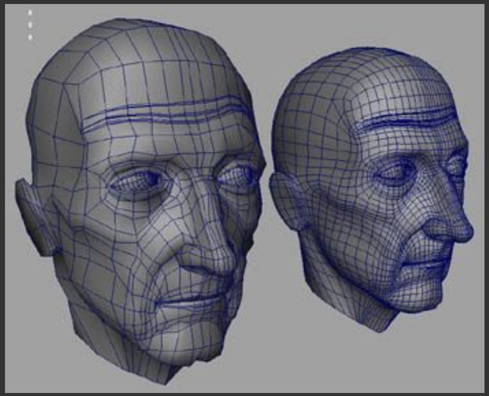
Controlled exposure

Complex lighting, media

Intricate geometric structure

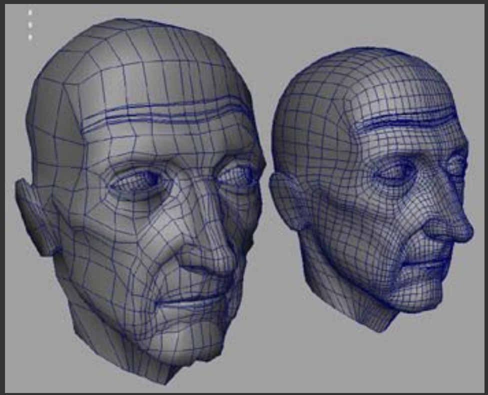
Diverse material appearances

Physically based image synthesis



Object representation

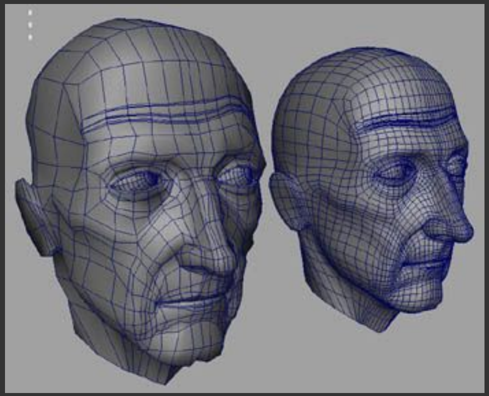
Physically based image synthesis



Object representation

Light transport

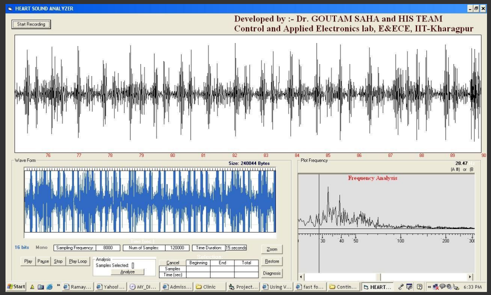
Physically based image synthesis



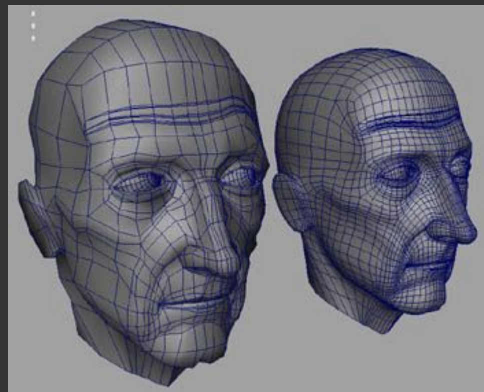
Object representation

Light transport

Physically based image synthesis



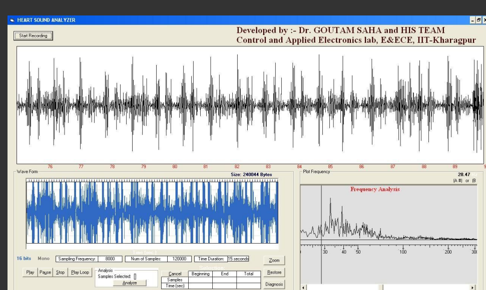
Digital signal processing



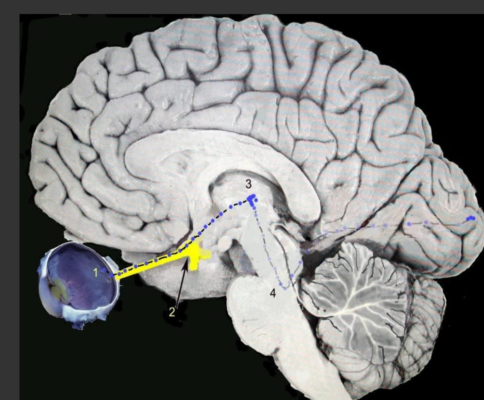
Object representation

Light transport

Physically based image synthesis

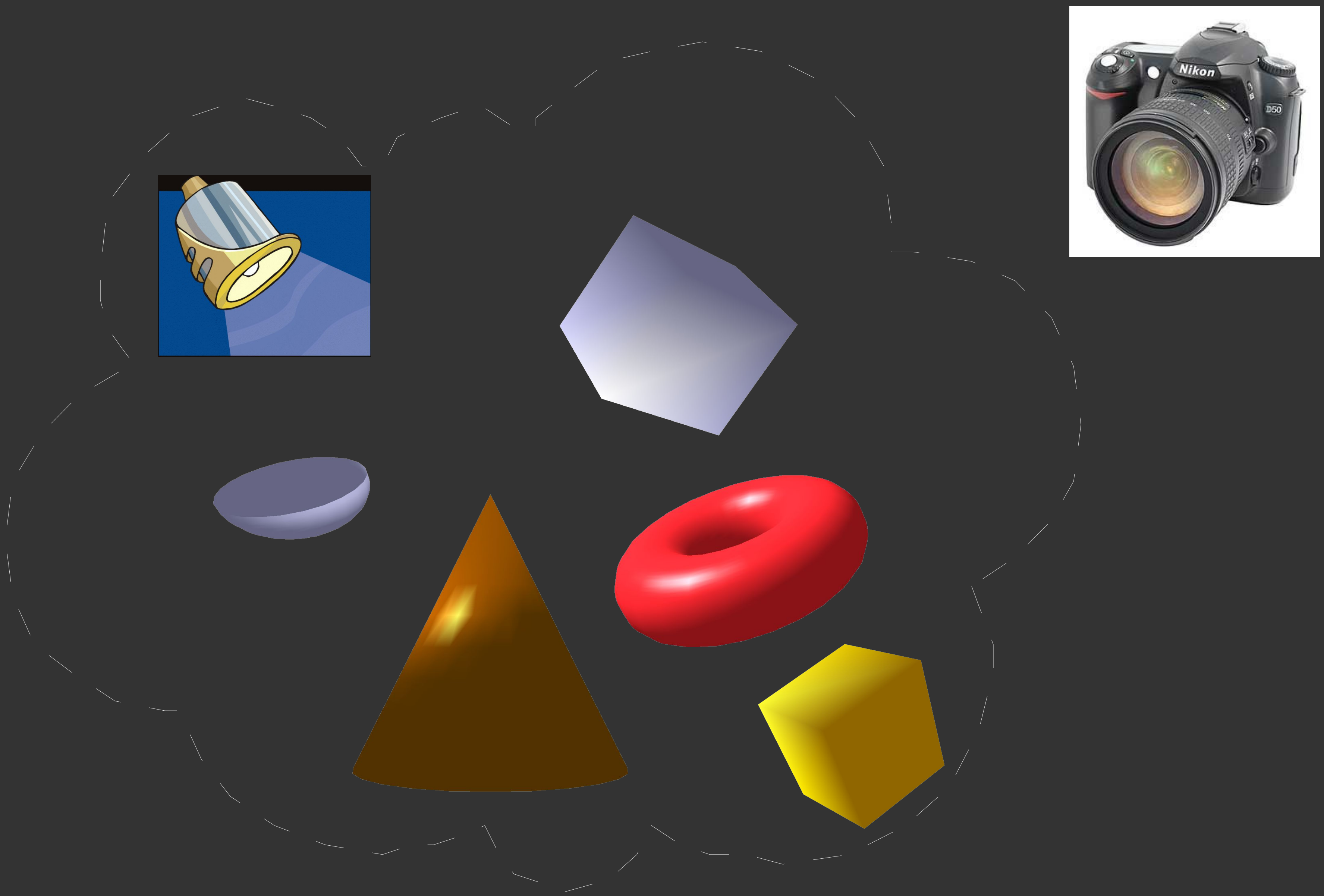


Digital signal processing

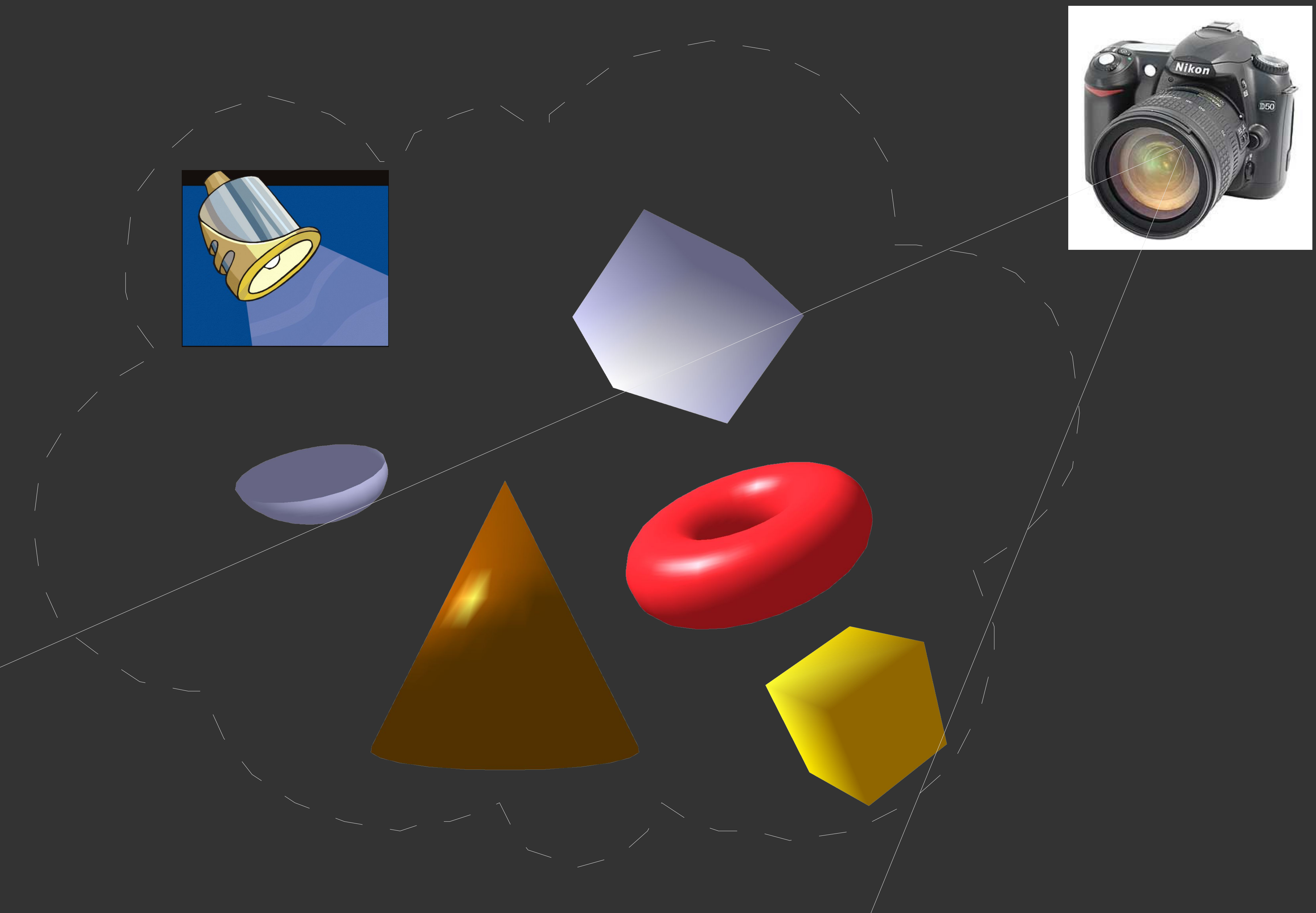


Human visual system

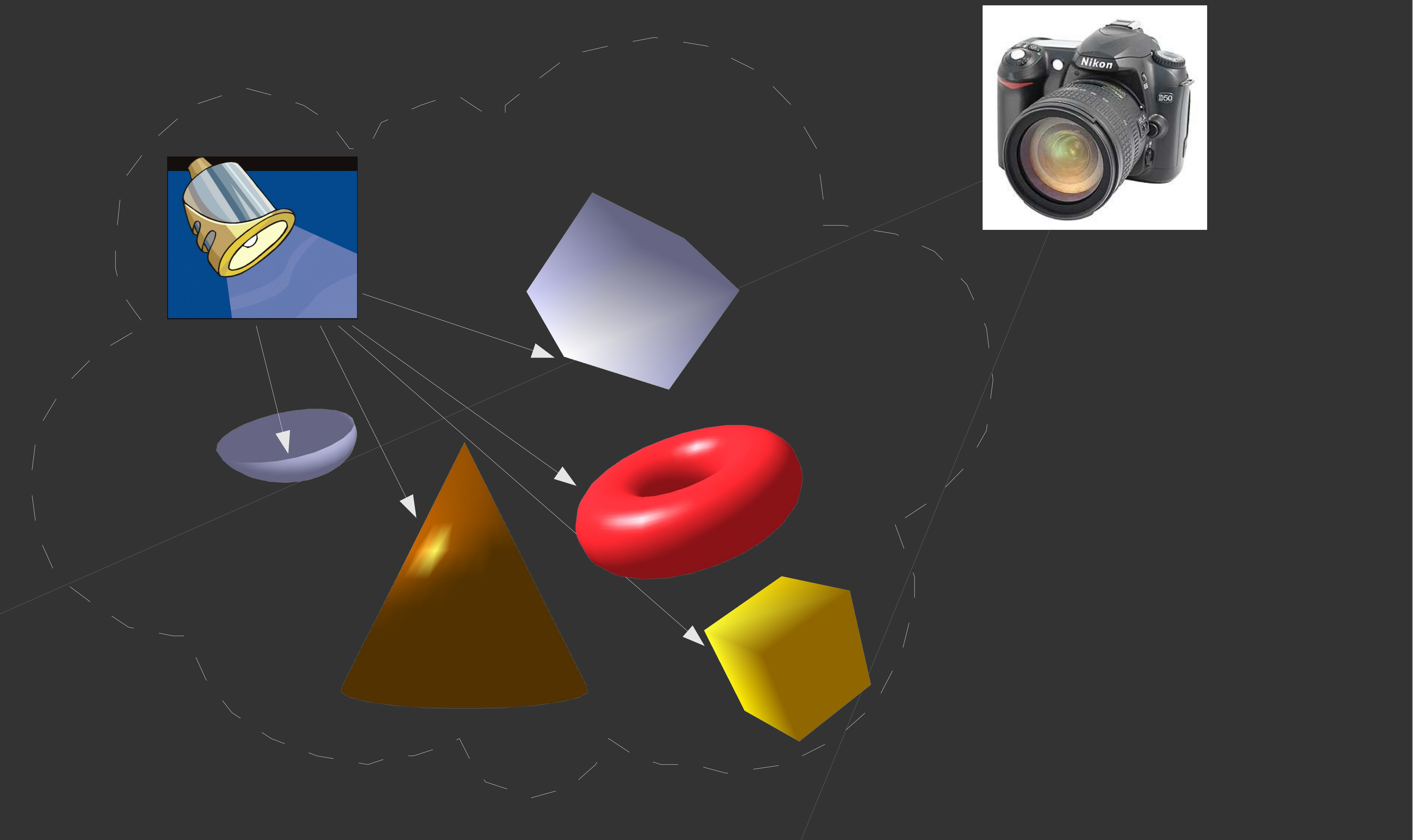
Light transport



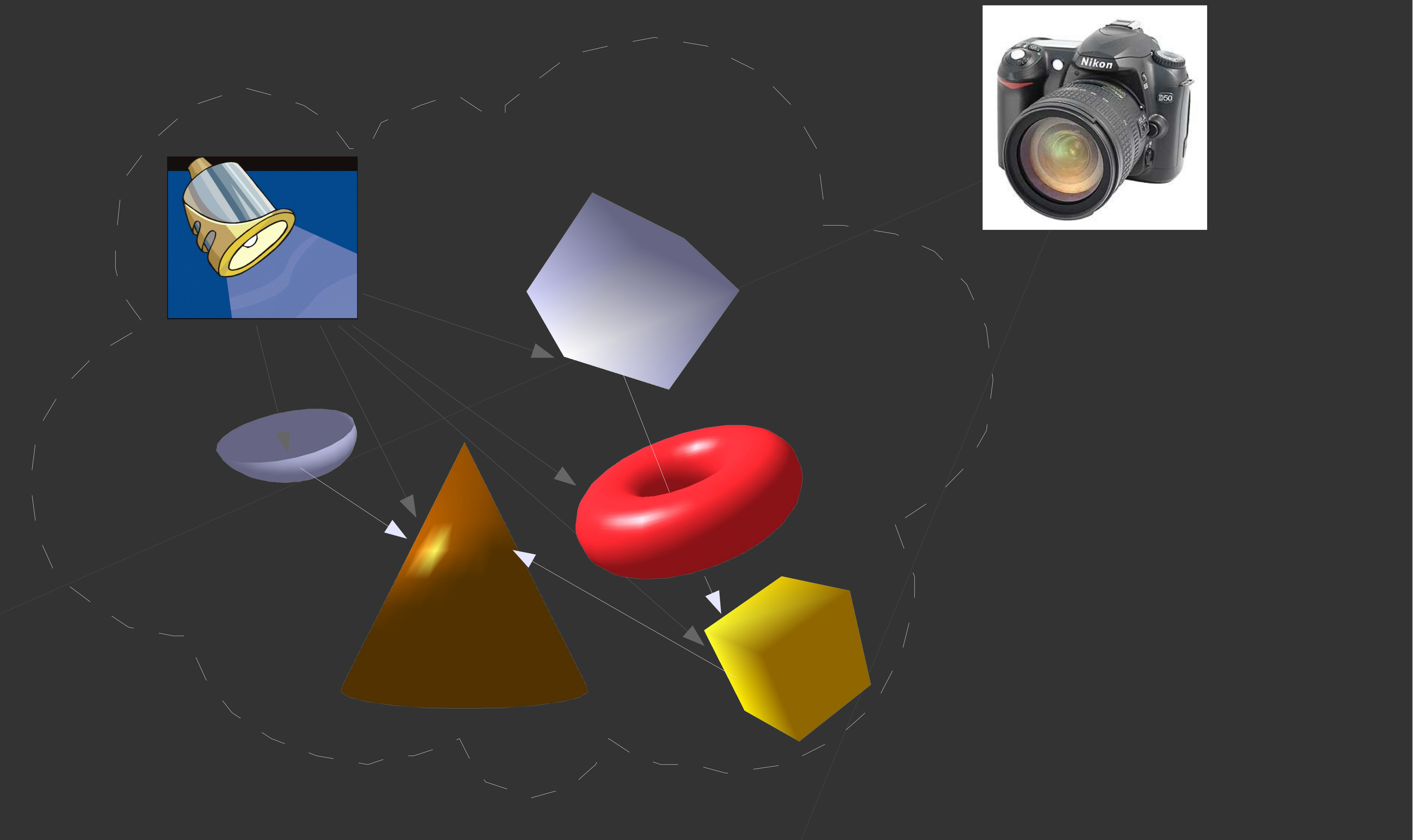
Light transport



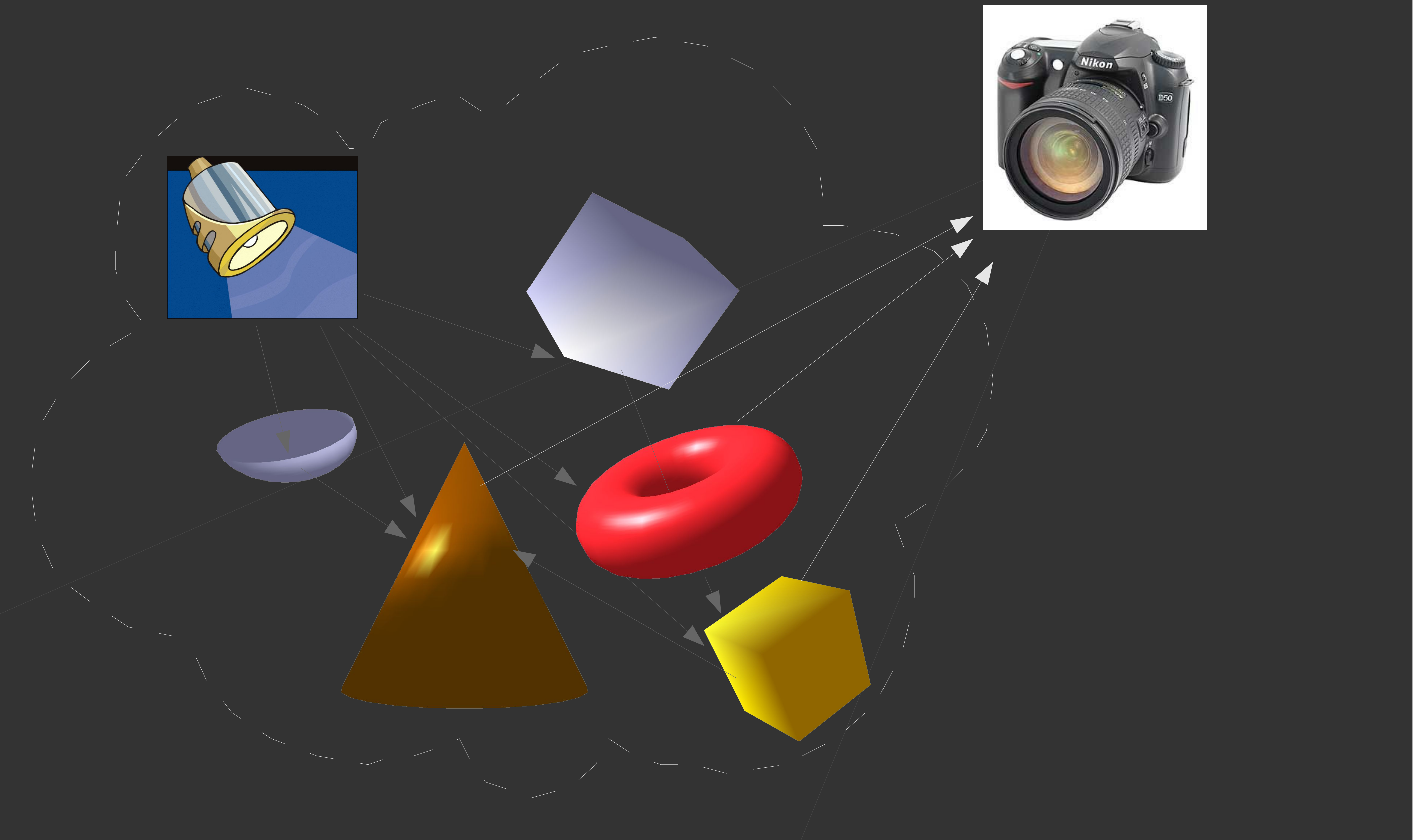
Light transport



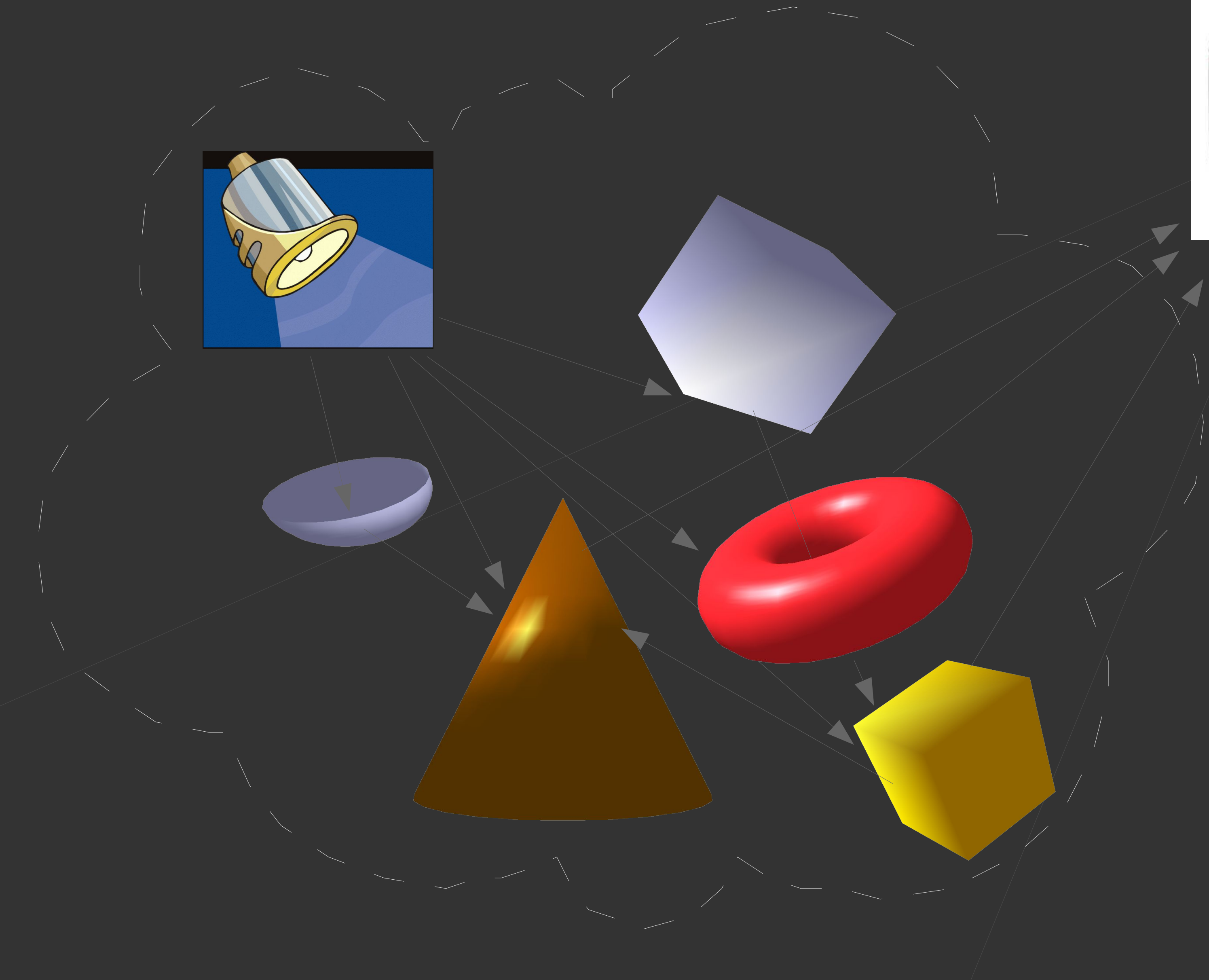
Light transport



Light transport

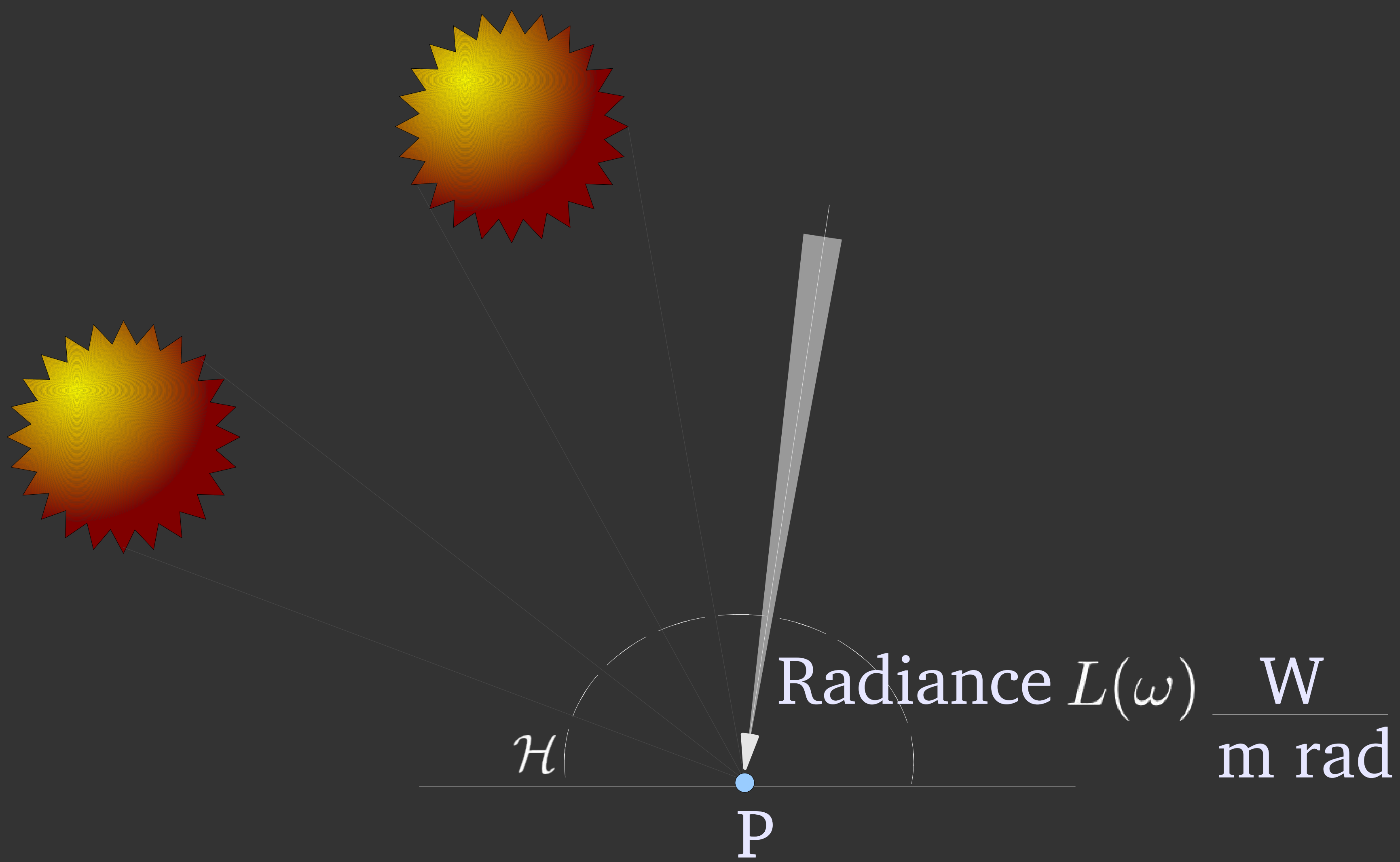


High complexity!



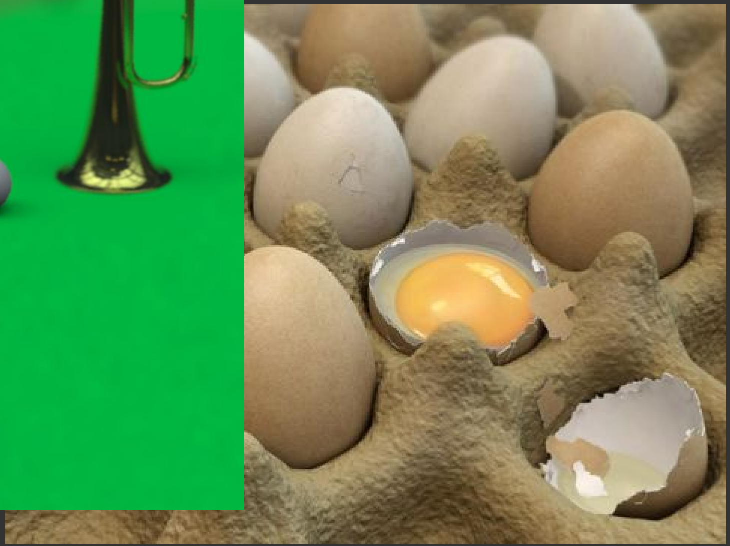
- Interreflection
- Refraction
- Occlusion
- Dispersion
- ...

Incident Illumination at P



Monte Carlo path tracing

- Image synthesis by tracing light paths
- Integrating over several domains



Monte Carlo path tracing ⟶ sampling

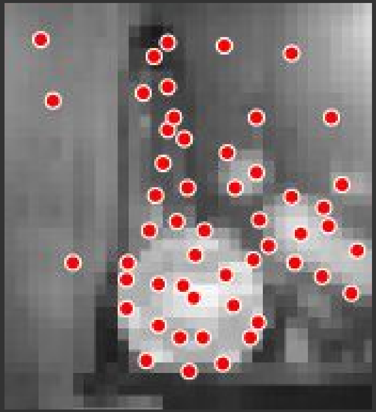
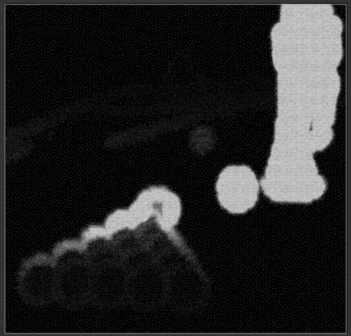
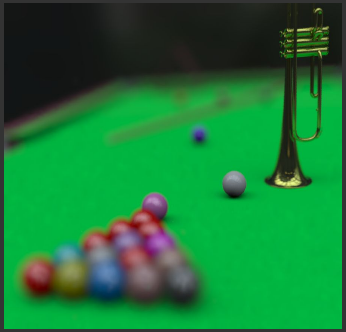


Image space



Estimate radiance at sample locations

Reconstruct image



Monte Carlo path tracing ⟶ sampling

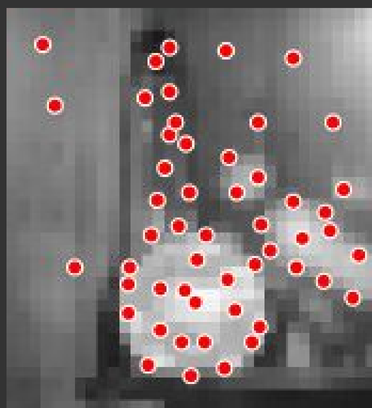
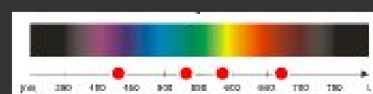
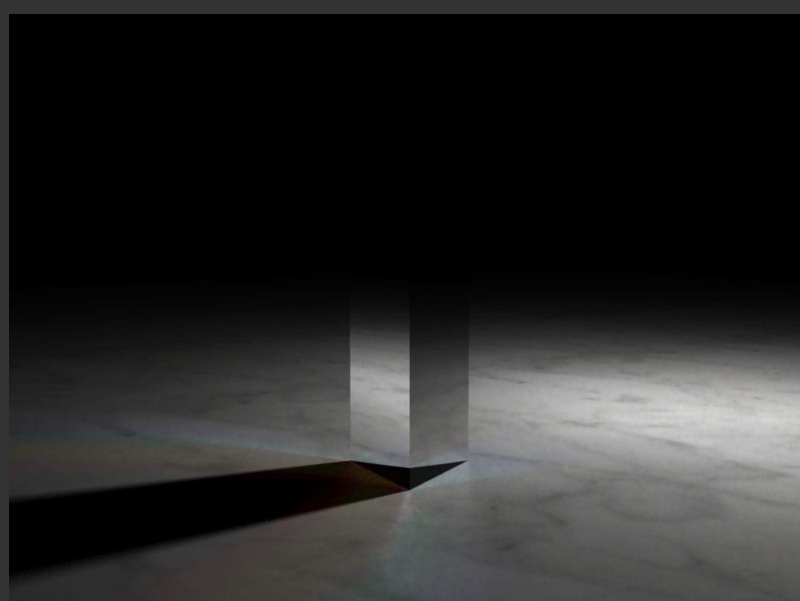


Image space



Visible spectrum



Monte Carlo path tracing ⟶ sampling

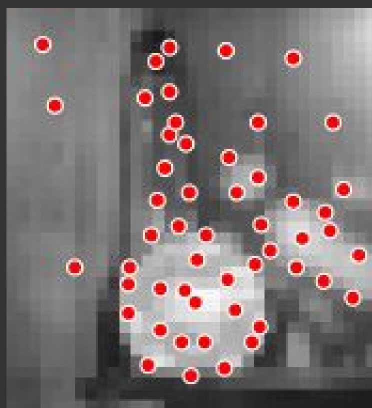
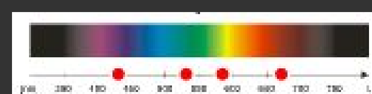


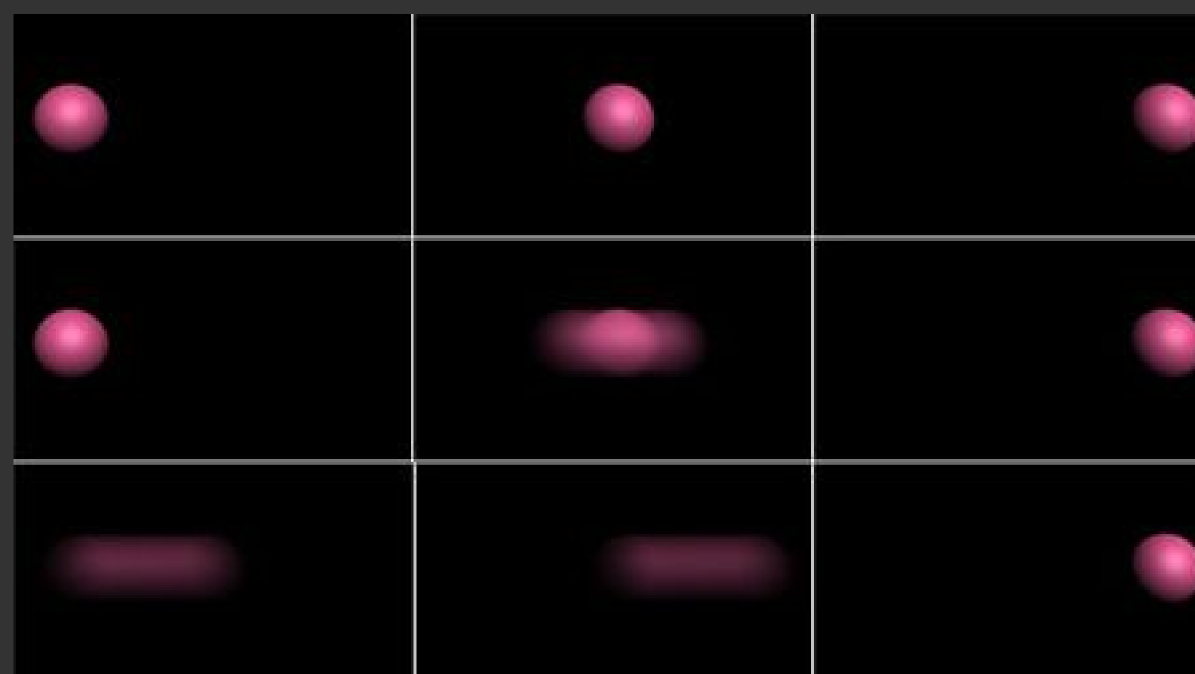
Image space



Visible spectrum

Aperture

Exposure time



Monte Carlo path tracing ⟶ sampling

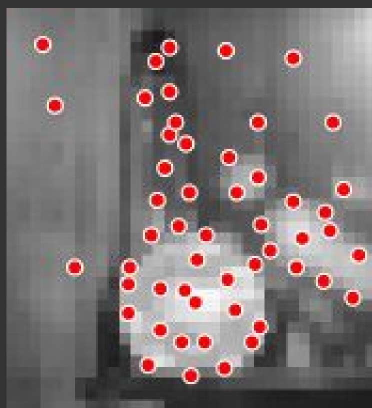
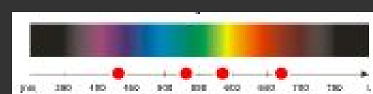


Image space



Visible spectrum

Aperture

Exposure time

Material reflectance functions

Direct illumination

Indirect illumination

Sampling Strategies for Efficient Image Synthesis

Kartic Subr

Monte Carlo path tracing $\longrightarrow$ *sampling*

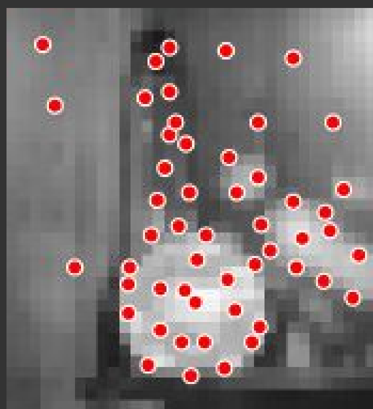
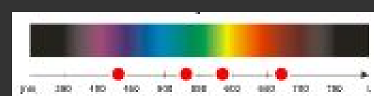


Image space



Visible spectrum

Aperture

Exposure time

Material reflectance functions

Direct illumination

Indirect illumination

Domains of interest

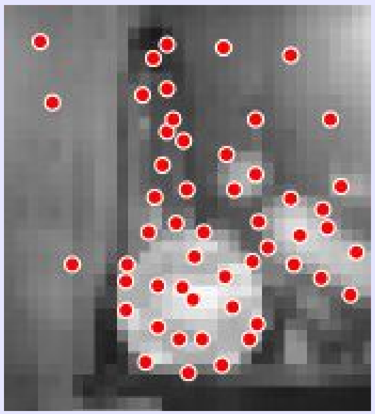
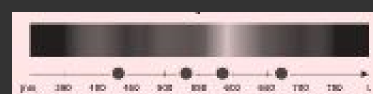


Image space



Visible spectrum

Aperture

Exposure time

Material reflectance functions

Direct illumination

Indirect illumination

Thesis

Thesis

1. Bandwidth prediction – depth of field simulation

Thesis

1. Bandwidth prediction – depth of field simulation
2. Steerable importance functions – direct distant illumination estimation

Thesis

1. Bandwidth prediction – depth of field simulation
2. Steerable importance functions – direct distant illumination estimation
3. Statistical hypotheses testing – assessing MC estimators

Questions ?

Questions ?

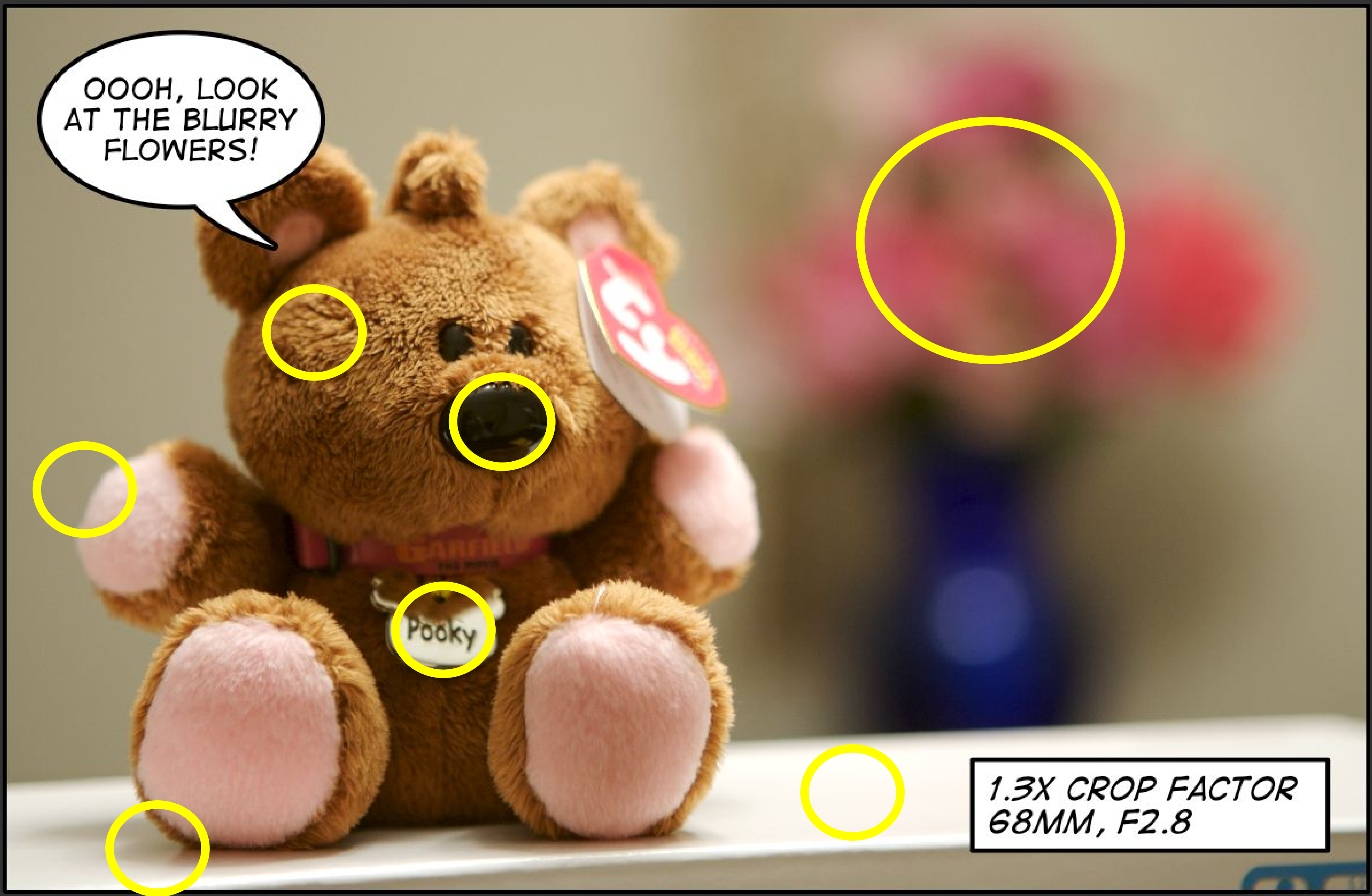
- Claims are true ? Says who ?

Bandwidth Prediction for Efficient Depth of Field Rendering

F. Durand, N. Holzschuch, F. Sillion, C. Soler, K. Subr

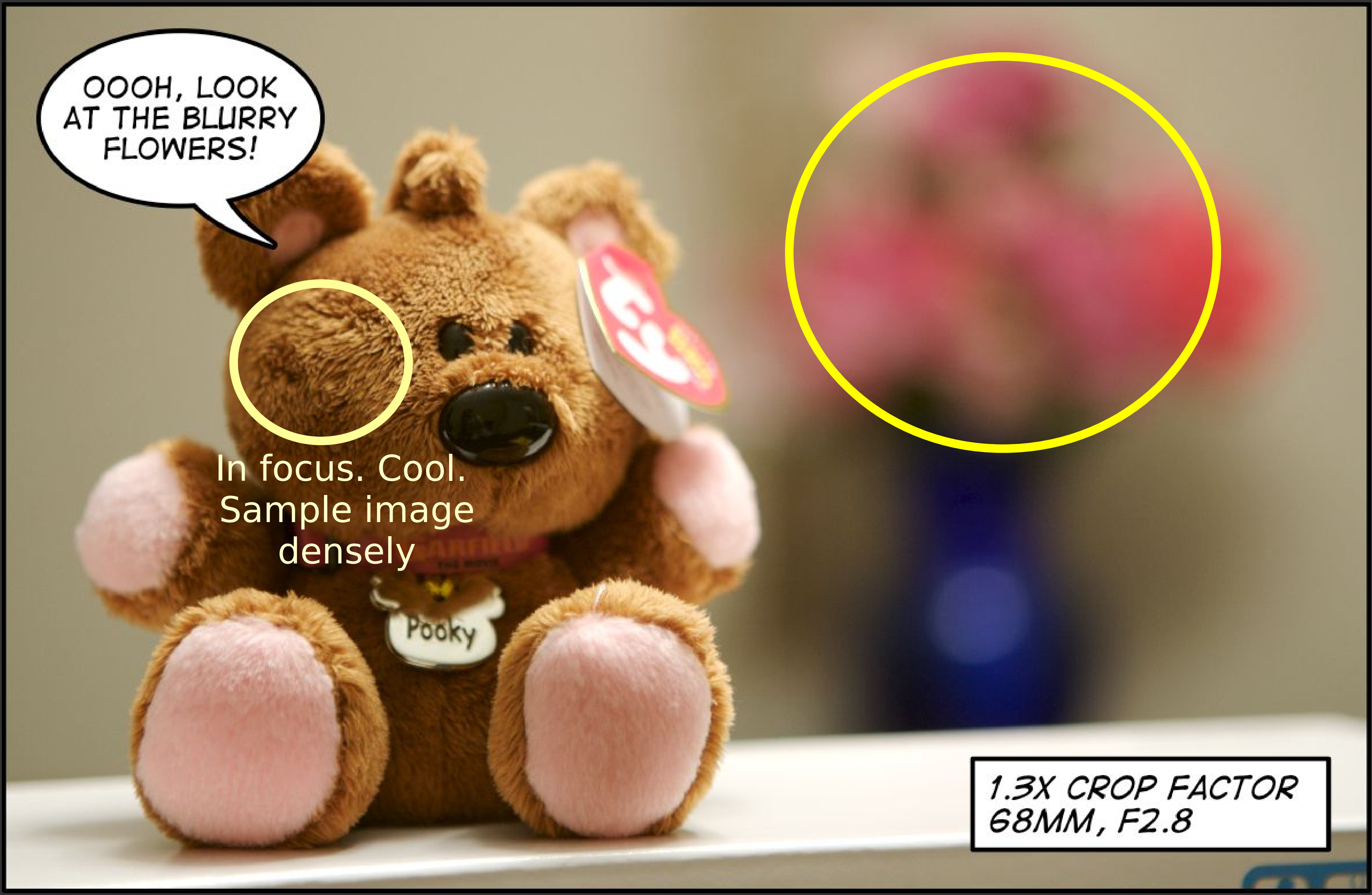

OOOH, LOOK AT THE BLURRY FLOWERS!
Pooky
1.3X CROP FACTOR
68MM, F2.8

Different Phenomena

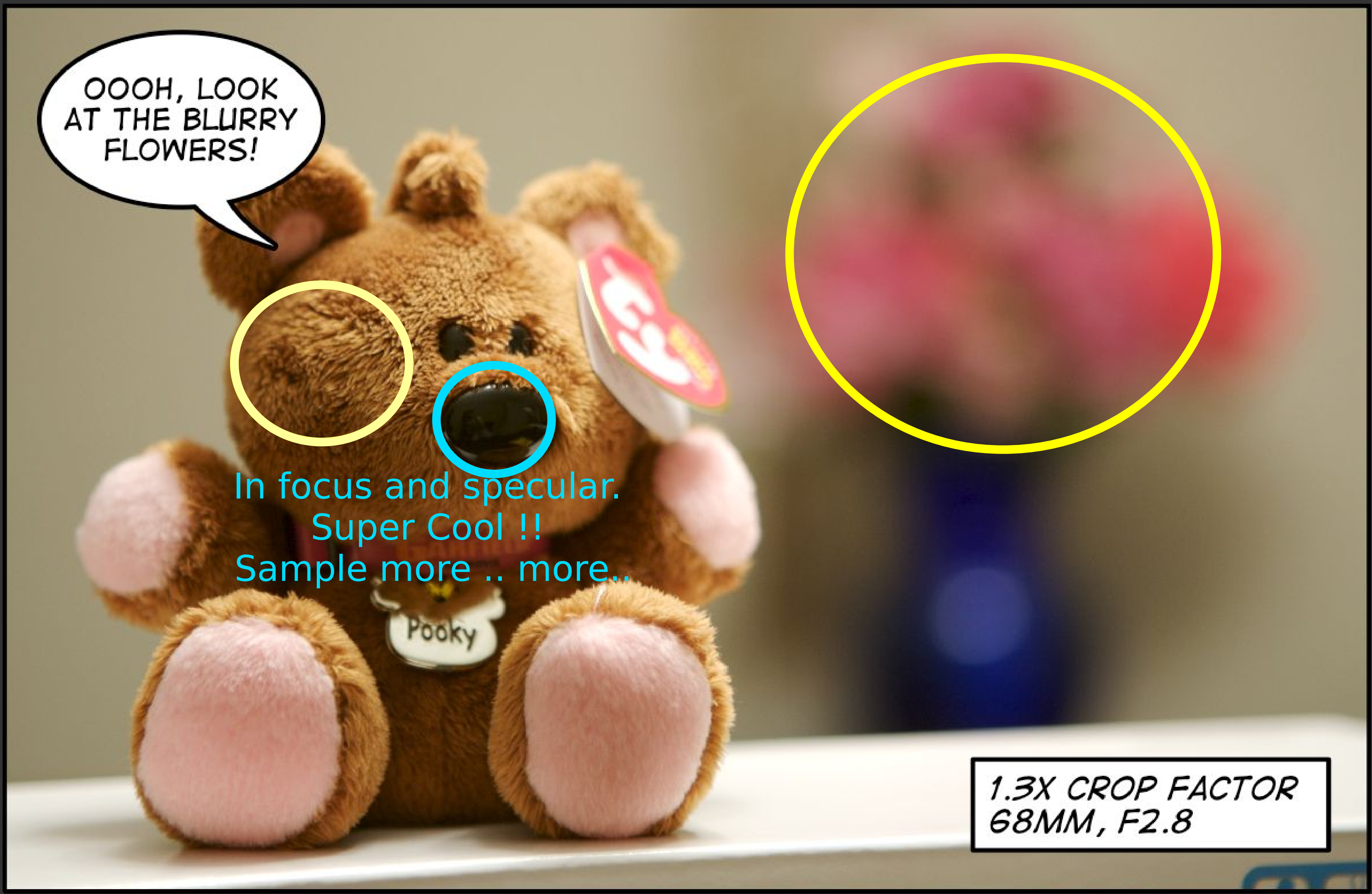


Depth Of Field

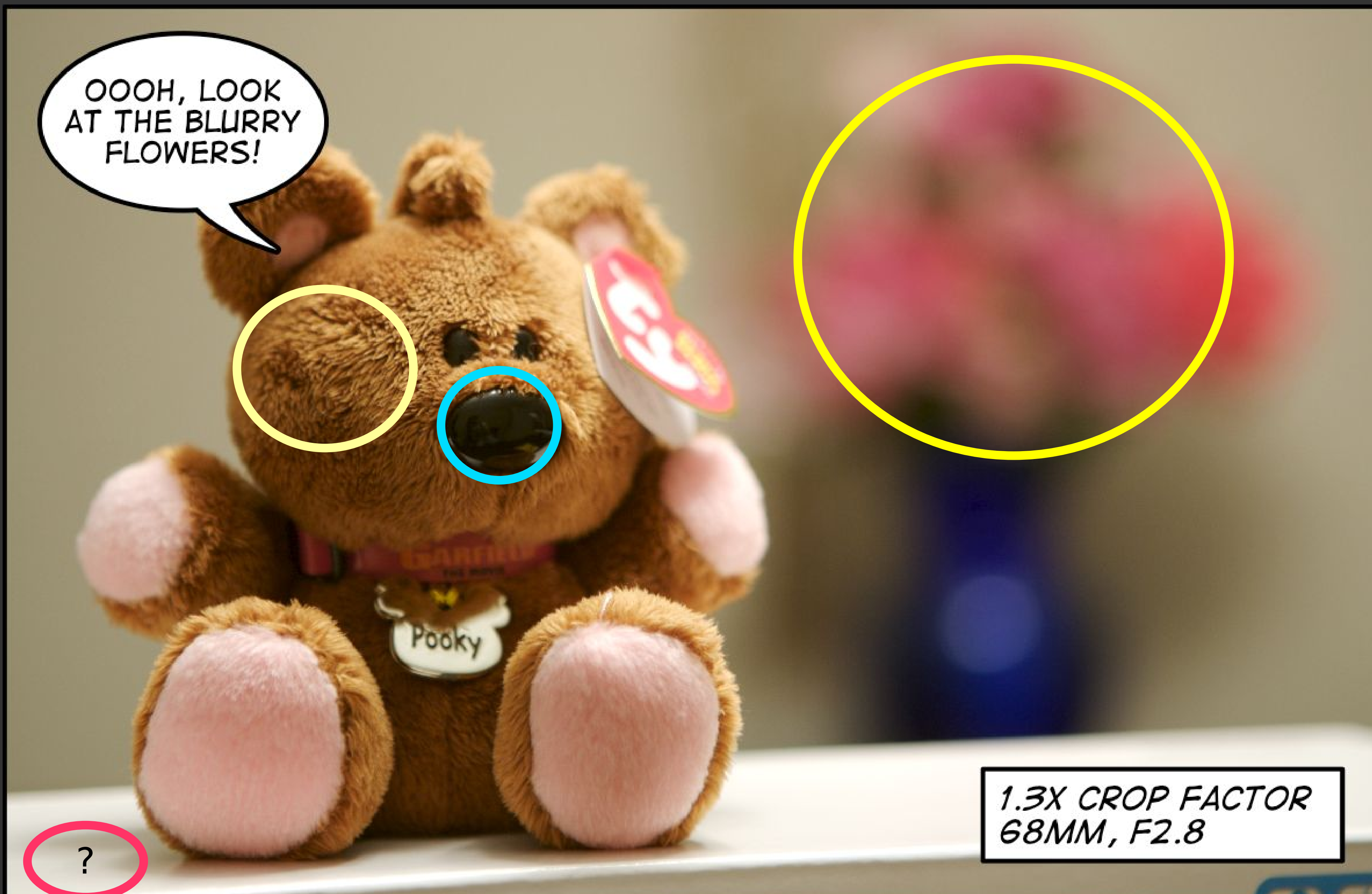
Depth Of Field



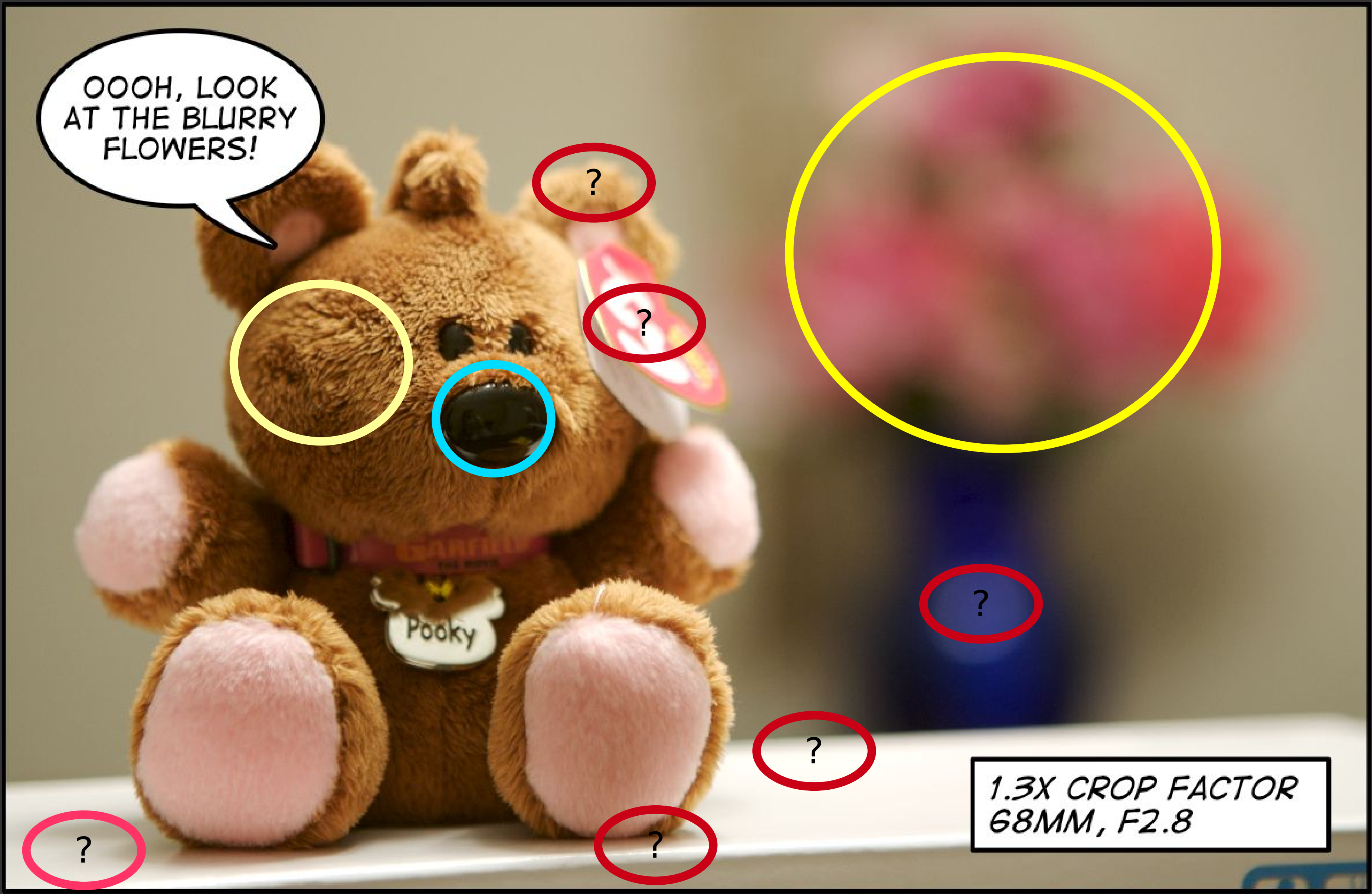
Specular Surfaces



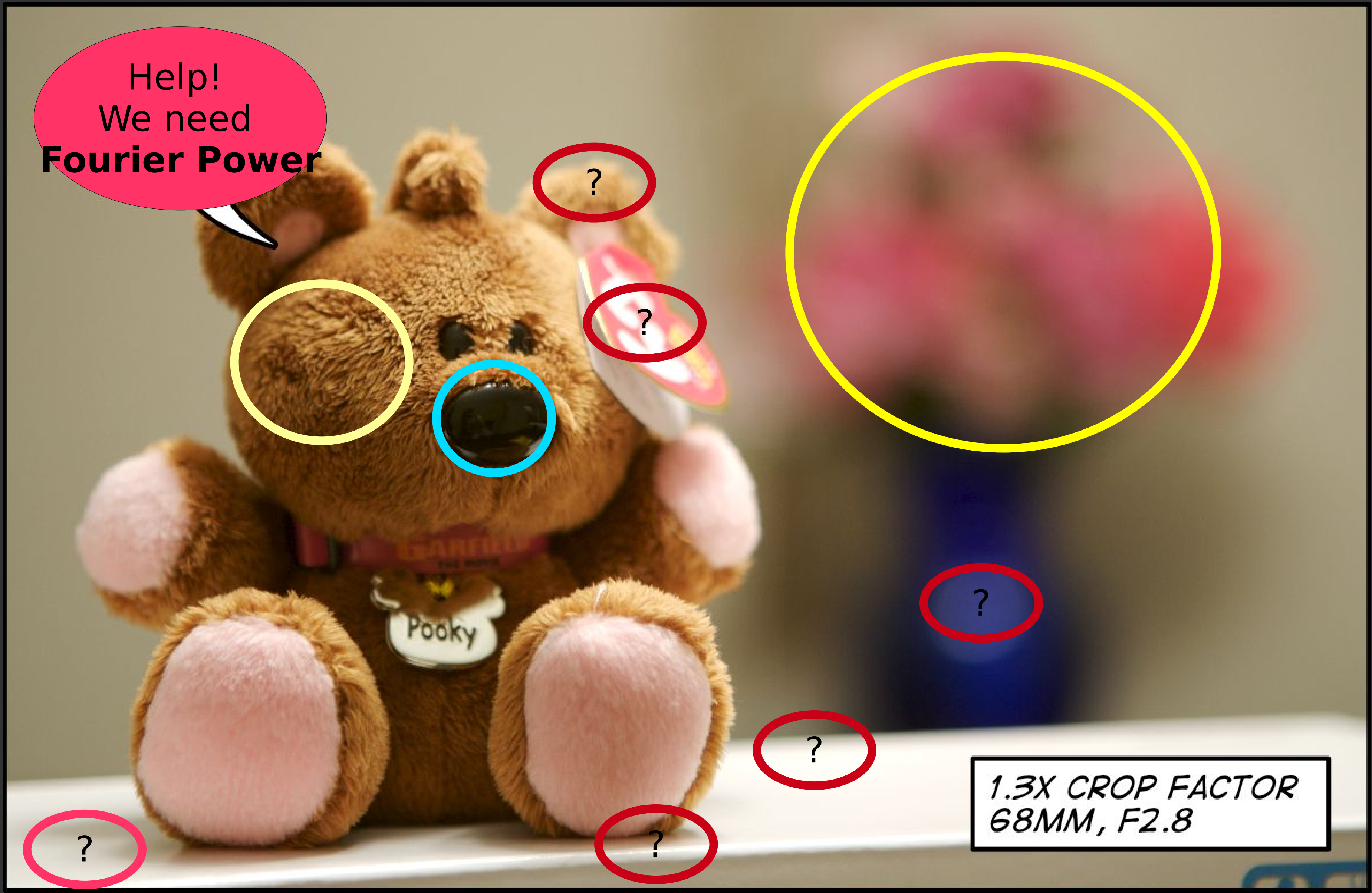
Soft Shadows



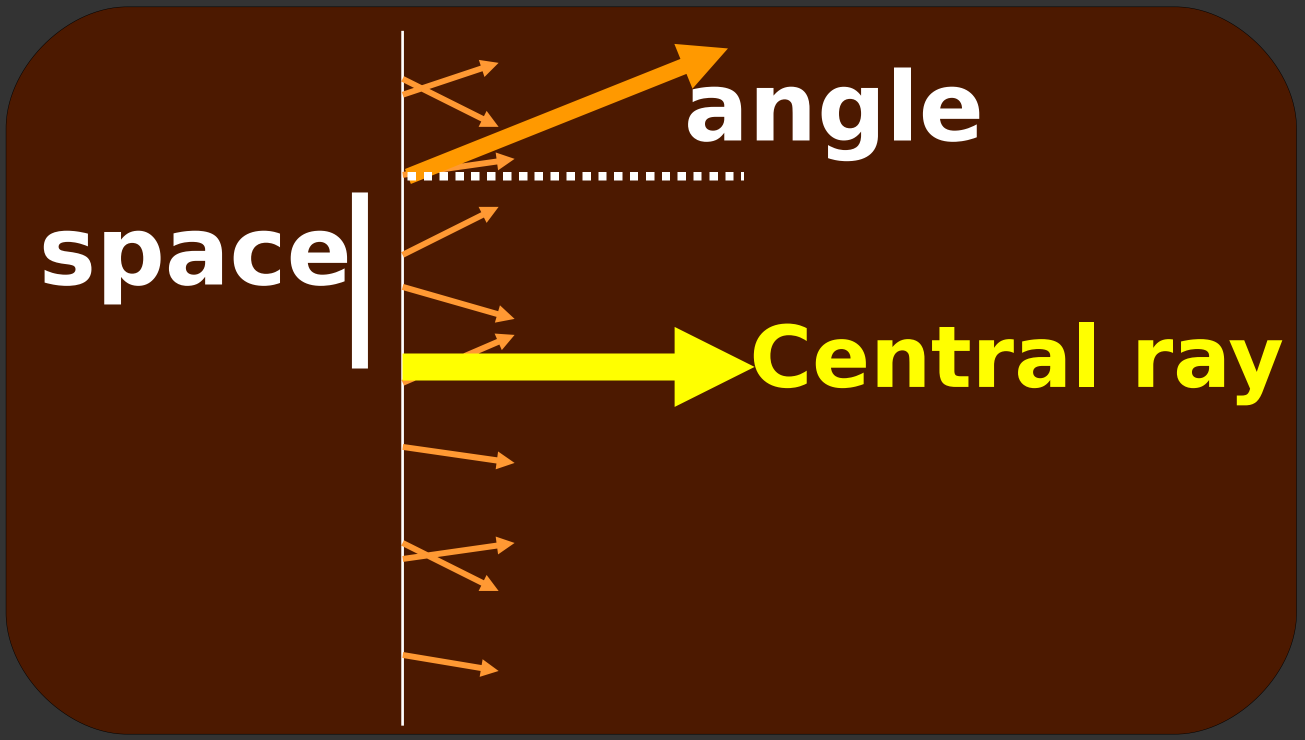
Combinations



Need for bandwidth prediction

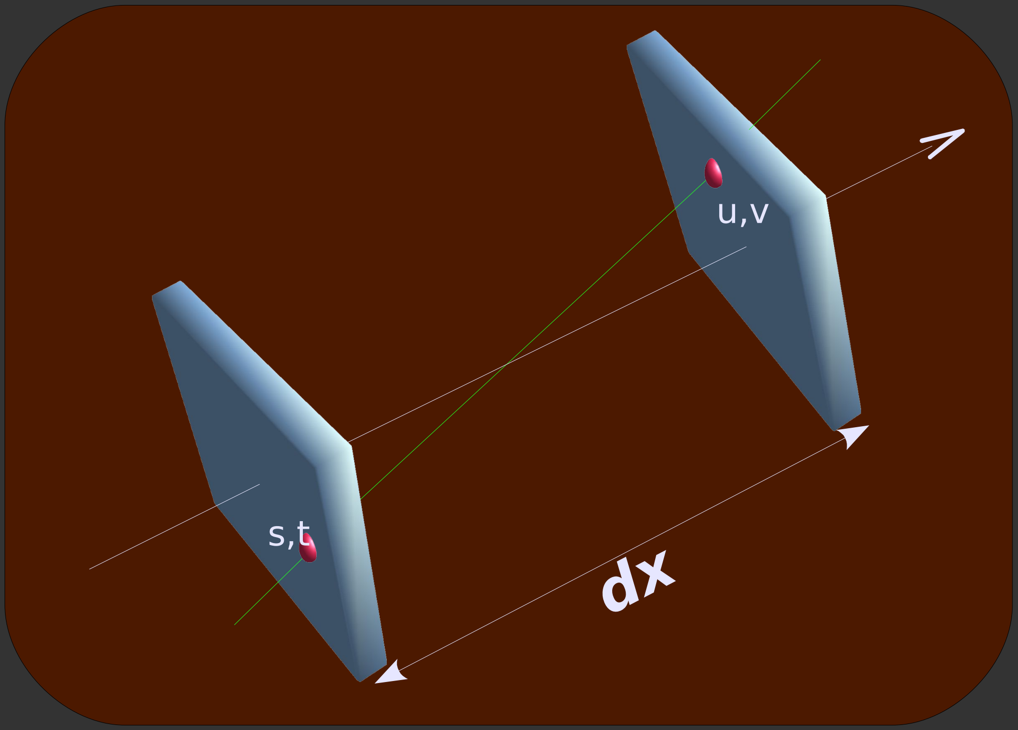


The 2D local light field



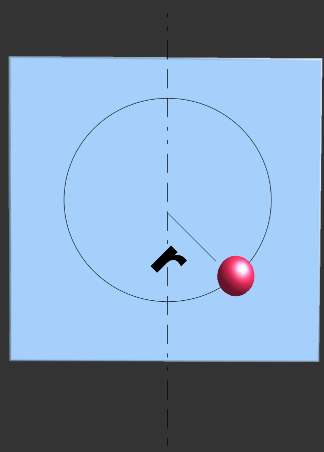
The 4D local light field

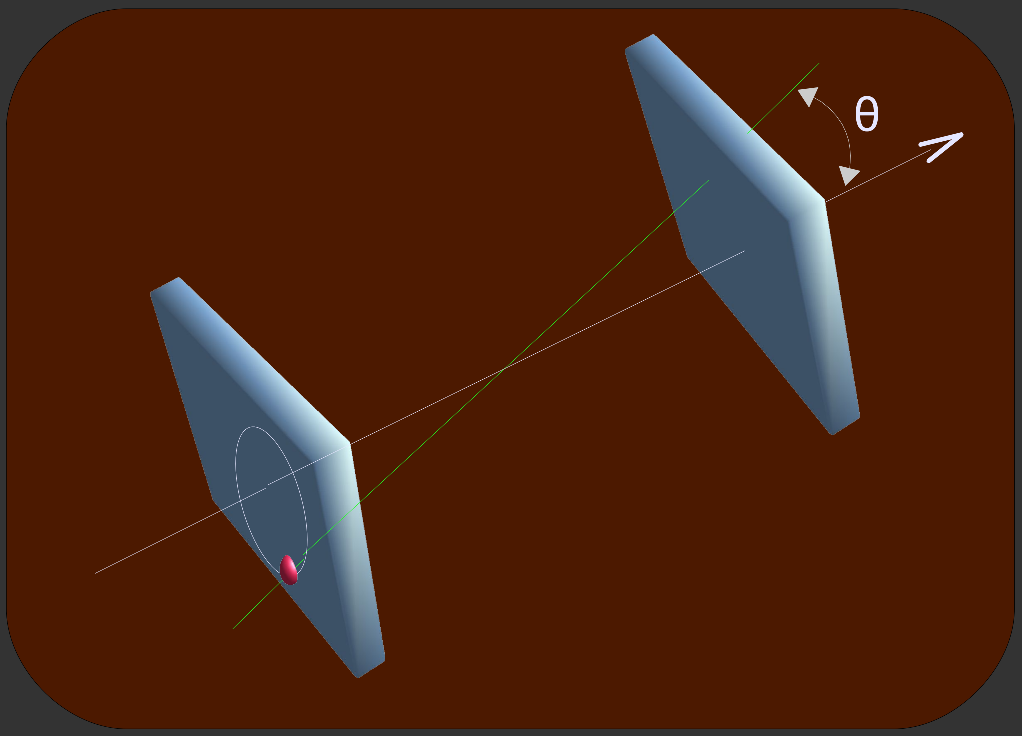
- Radiance along ray (s,t,u,v)



The 4D local light field

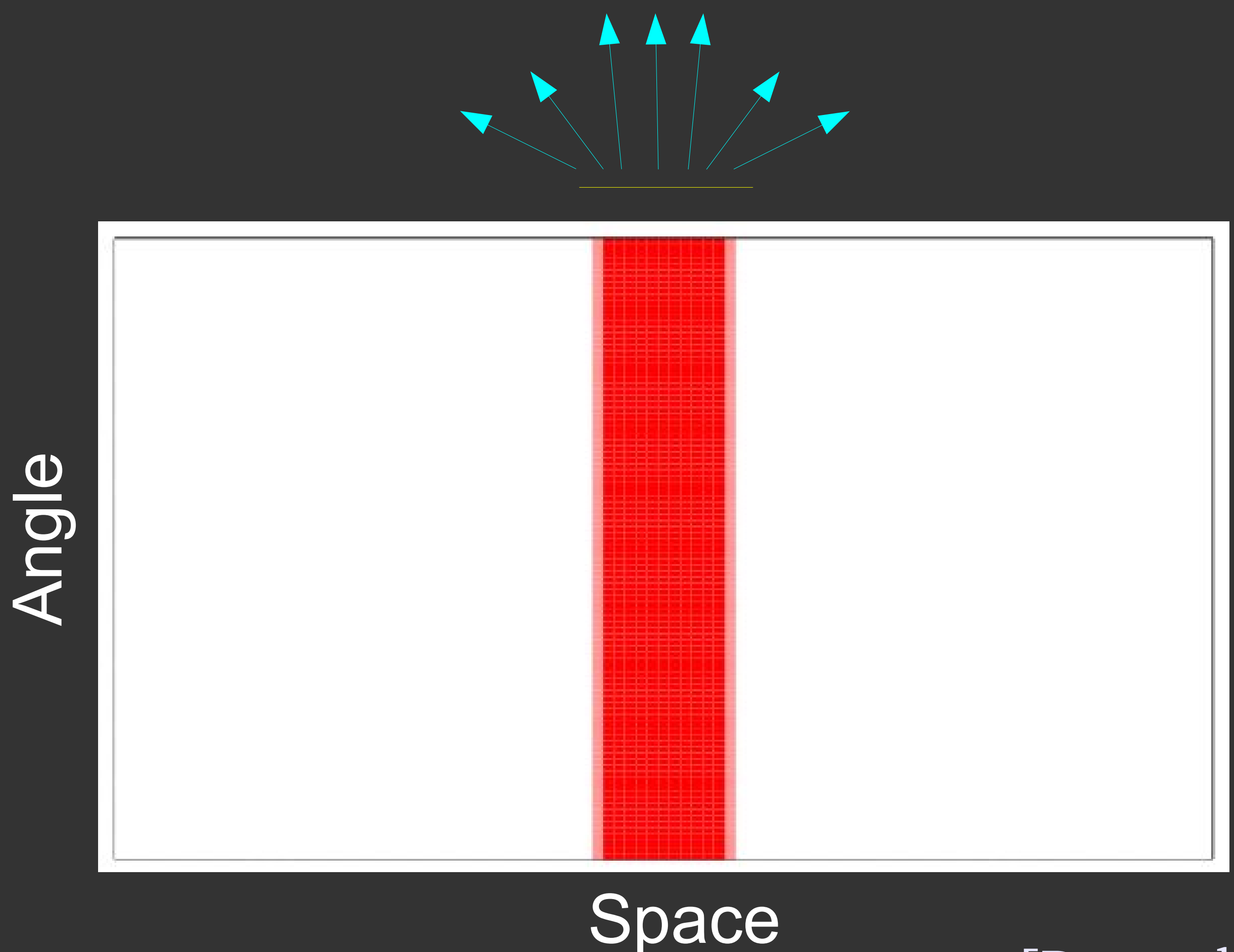
- Reduce to 2D – assume isotropy





Local Light Field Density Plot

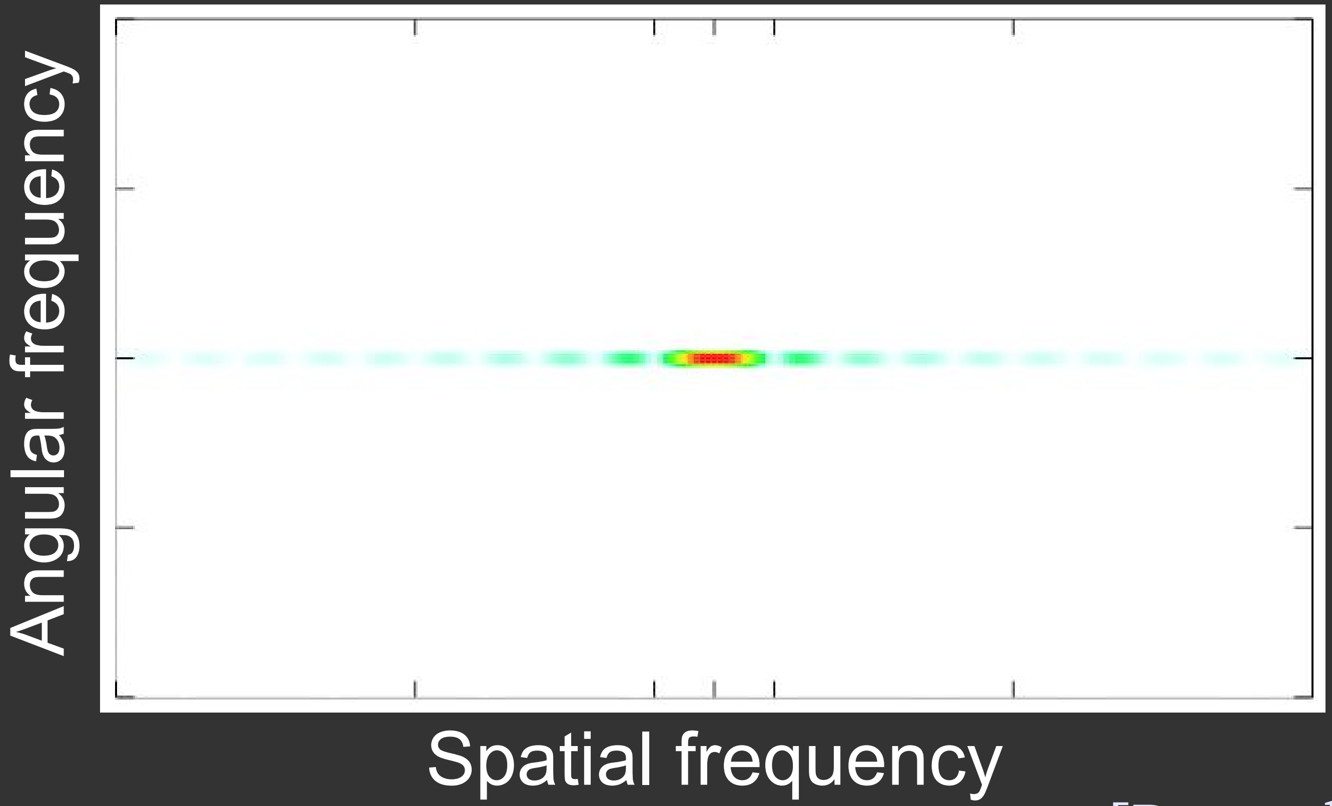
- On a 1D area emitter



[Durand et al. 2005]

Fourier Transform of Local Light Field

- Spatial: sinc
- Angular: Dirac delta



[Durand et al. 2005]

Spatial and Angular Frequencies

Hard Shadows
High Spatial Frequencies

Soft Shadows
Low Spatial Frequencies

Decreasing Angular Frequencies (left to right)

[Ramamoorthi and Hanrahan 2001]

Transport Phenomena- Summary

	Ray / Fourier space	Effect
Transport	Shear / Shear	
Occlusion	Multiplication/Convolution	Adds spatial frequencies
Reflection	Convolution/Multiplication	Removes angular frequencies
Curvature	Shear / Shear	

Transport Phenomena- Summary

	Ray / Fourier space	Effect
Transport	Shear / Shear	
Occlusion	Multiplication/Convolution	Adds spatial frequencies
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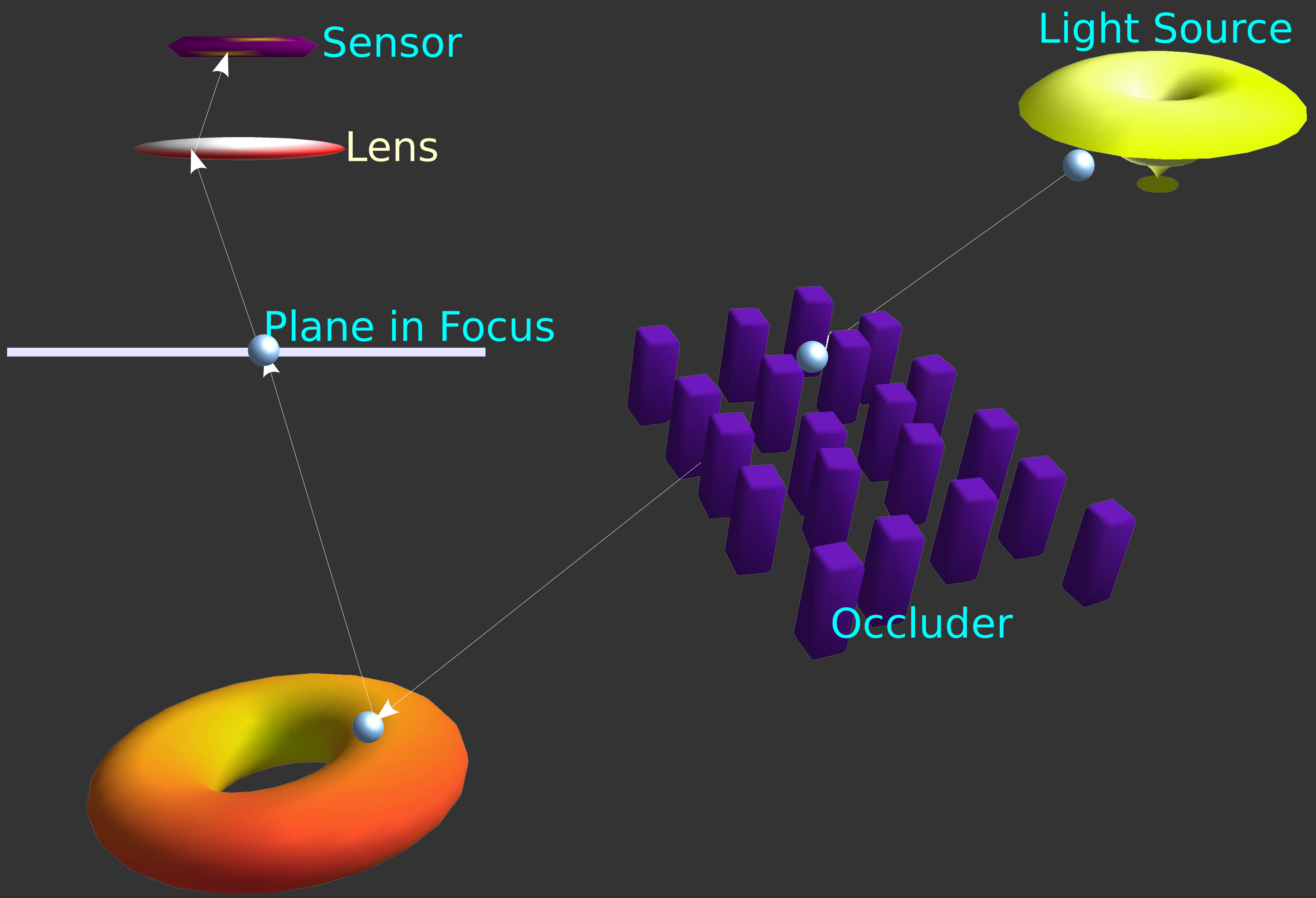
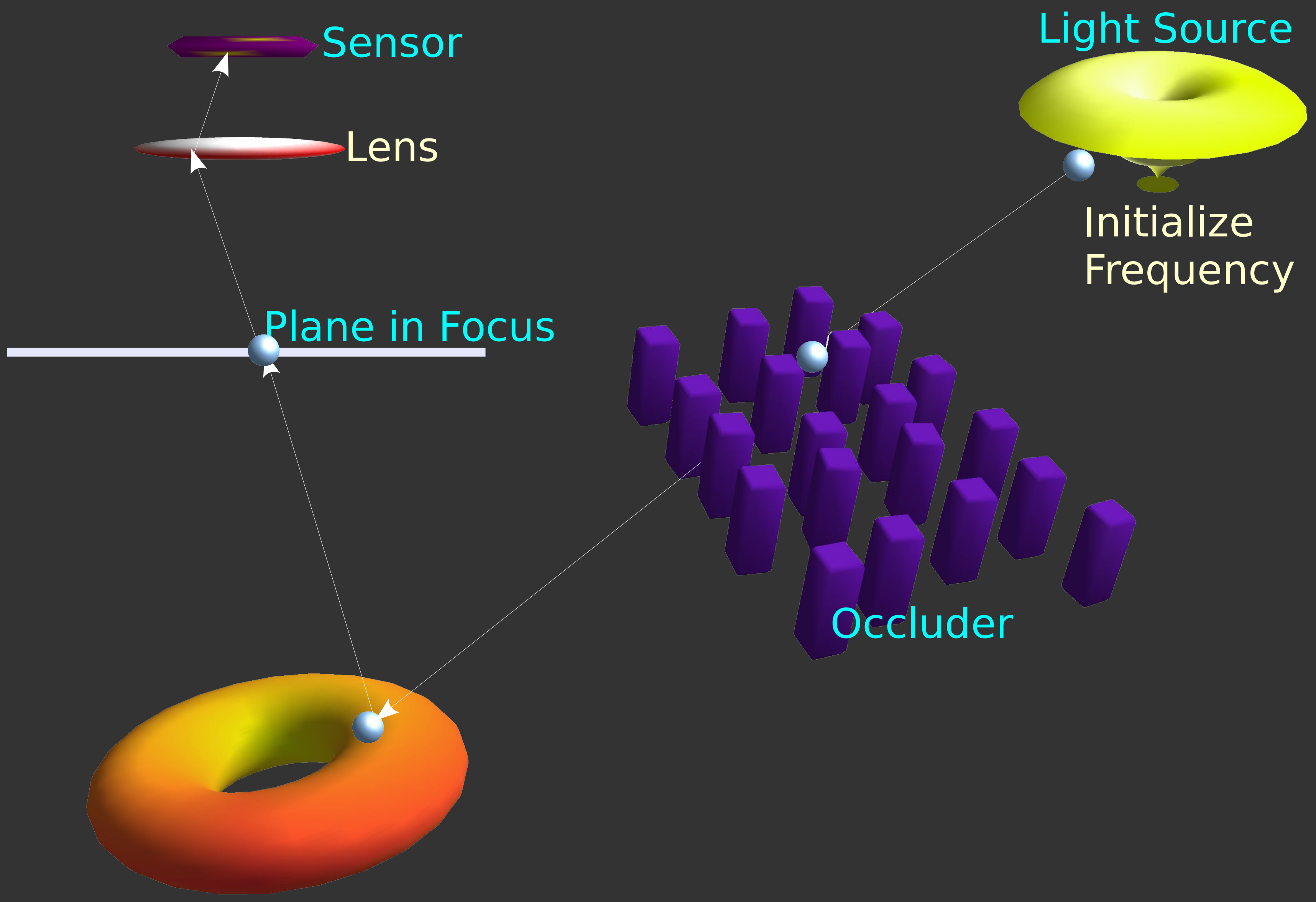
Curvature	Shear / Shear	

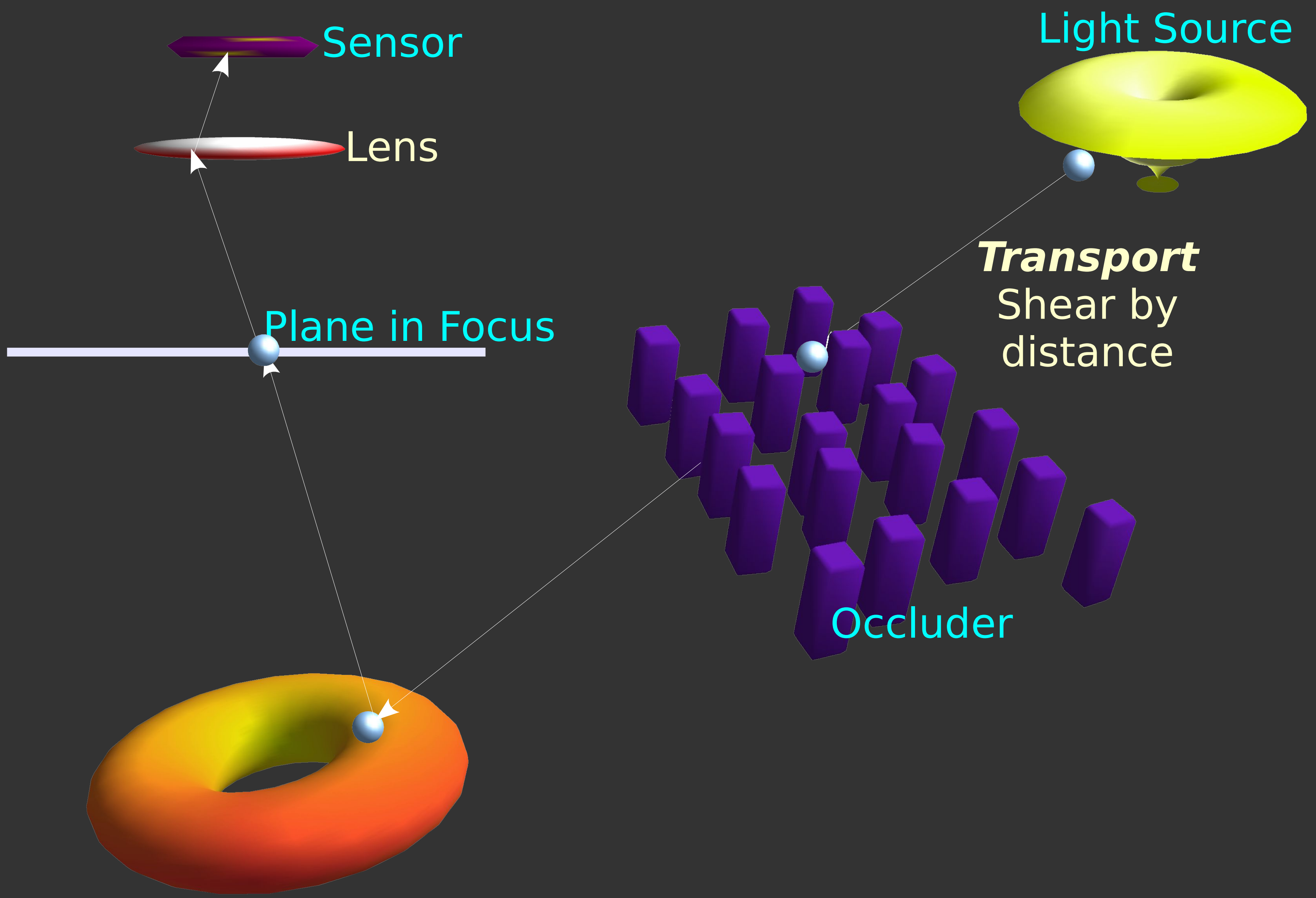
Frequency Transport



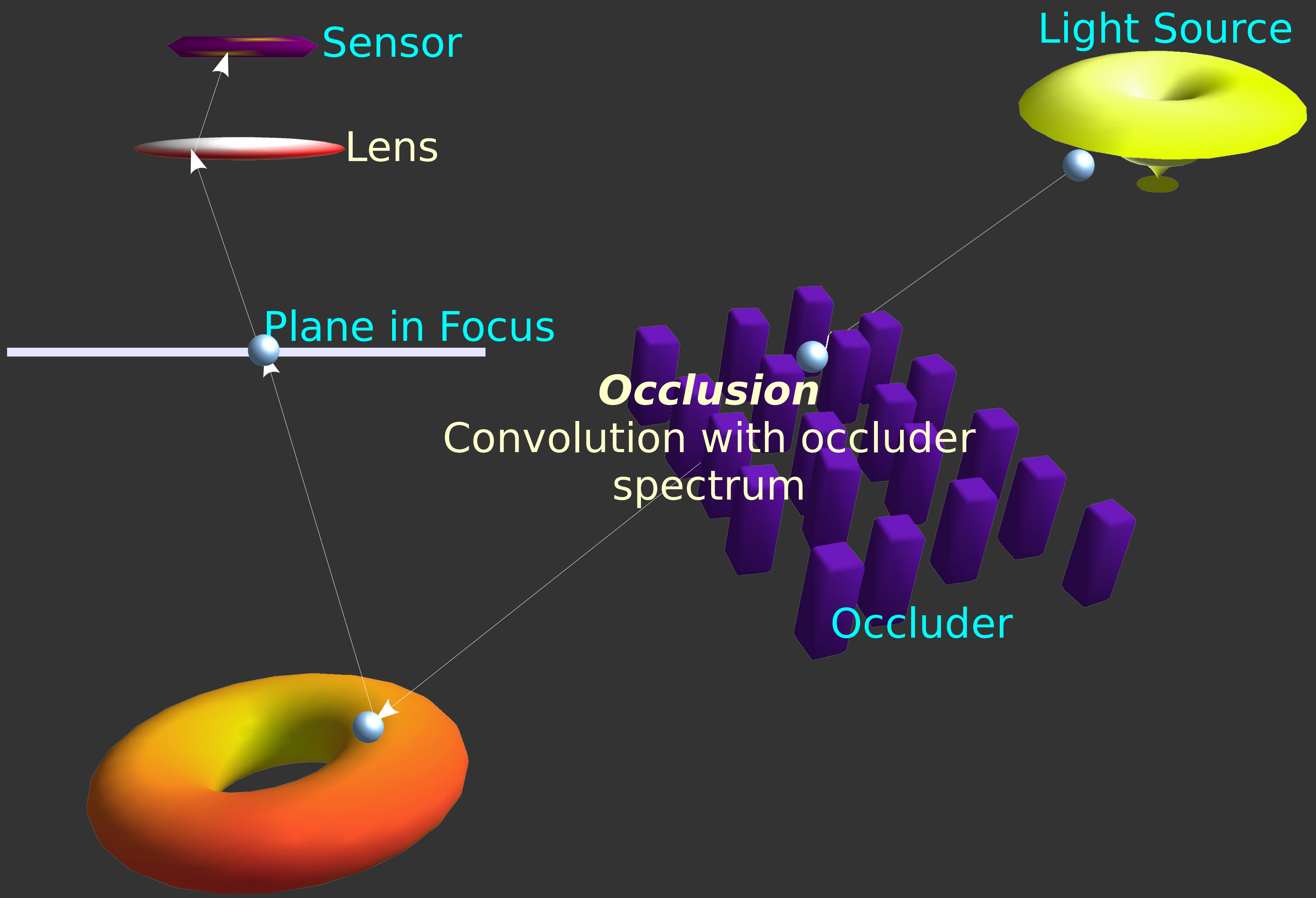
Frequency Transport



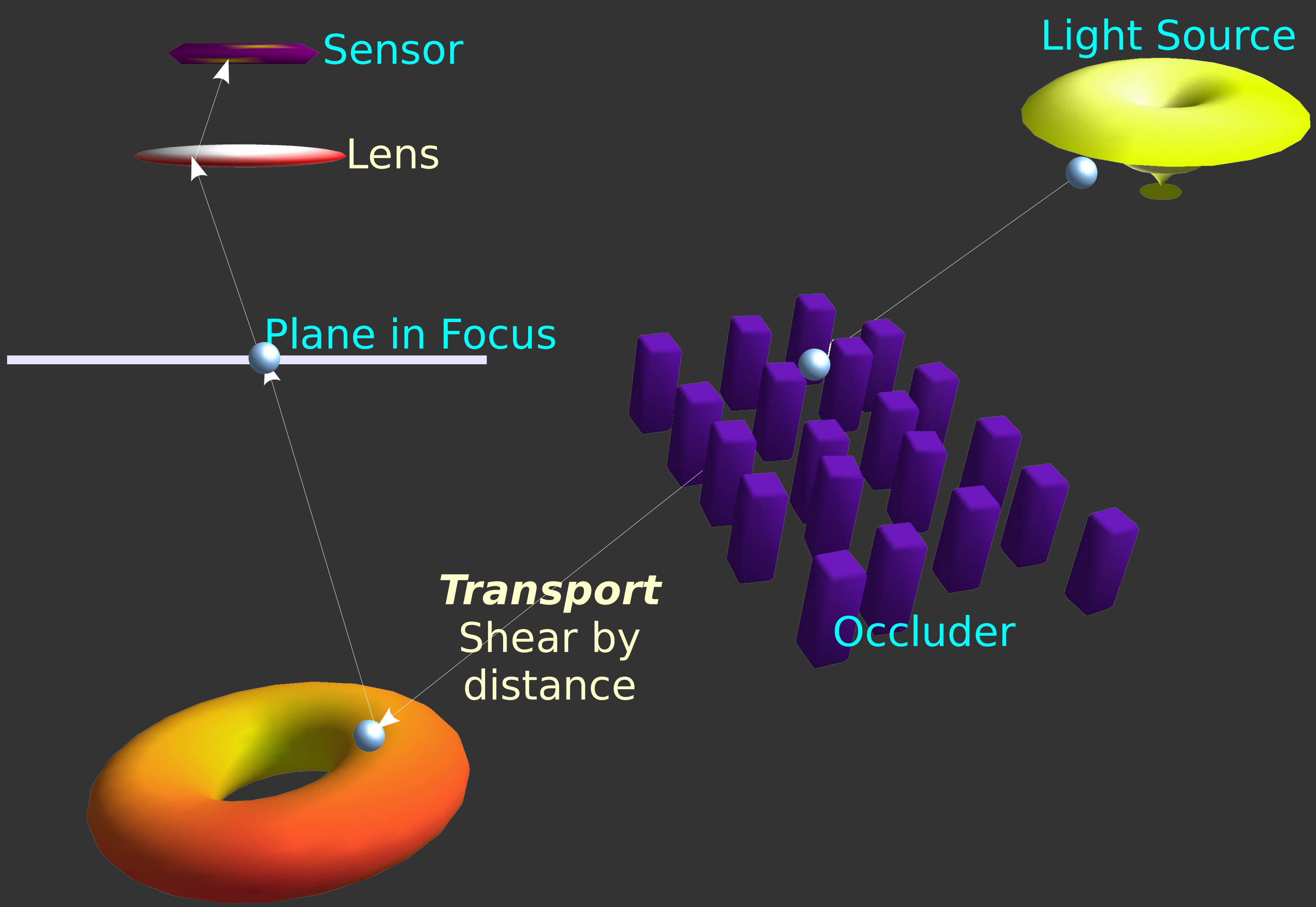
Frequency Transport



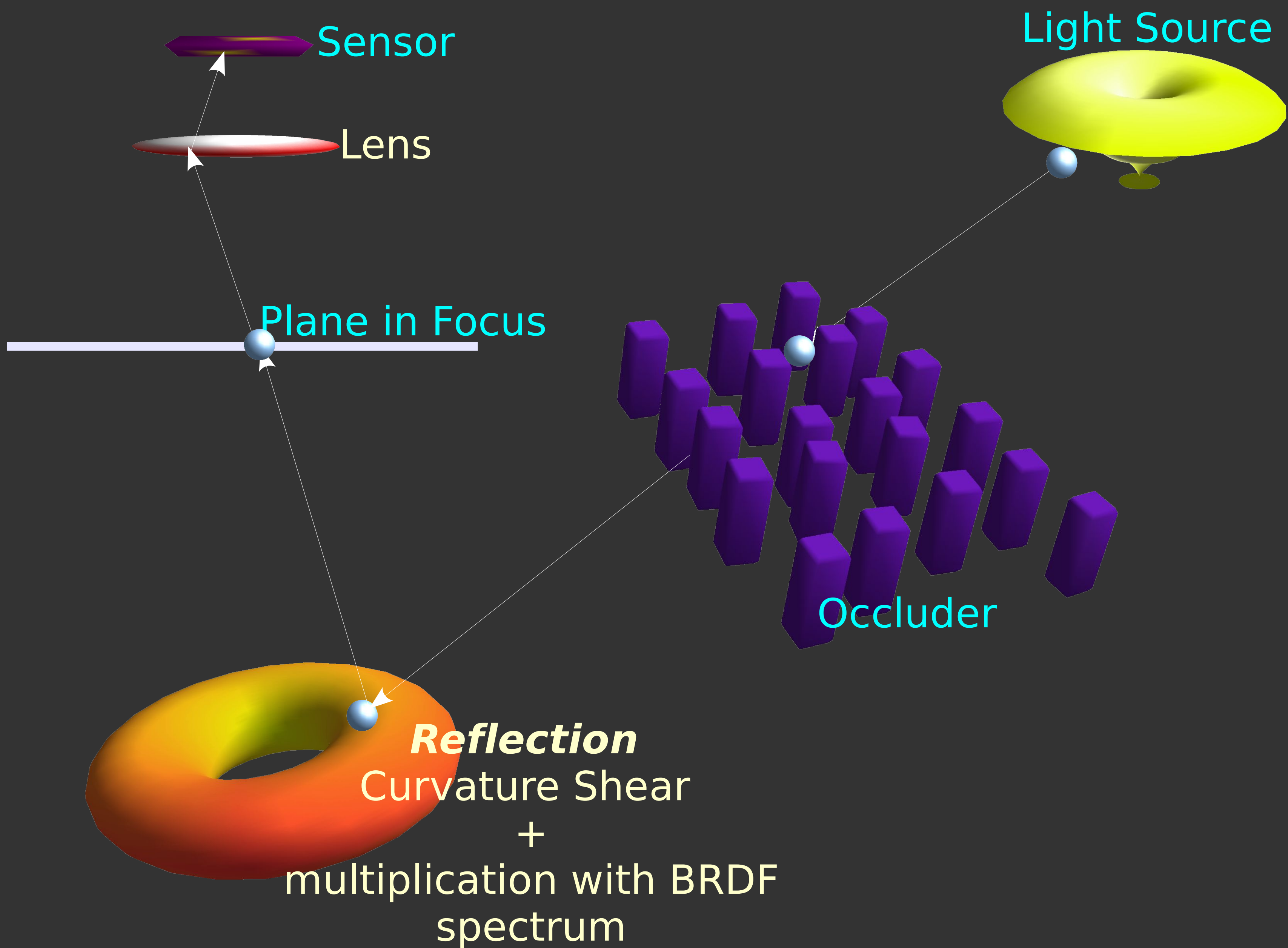
Frequency Transport



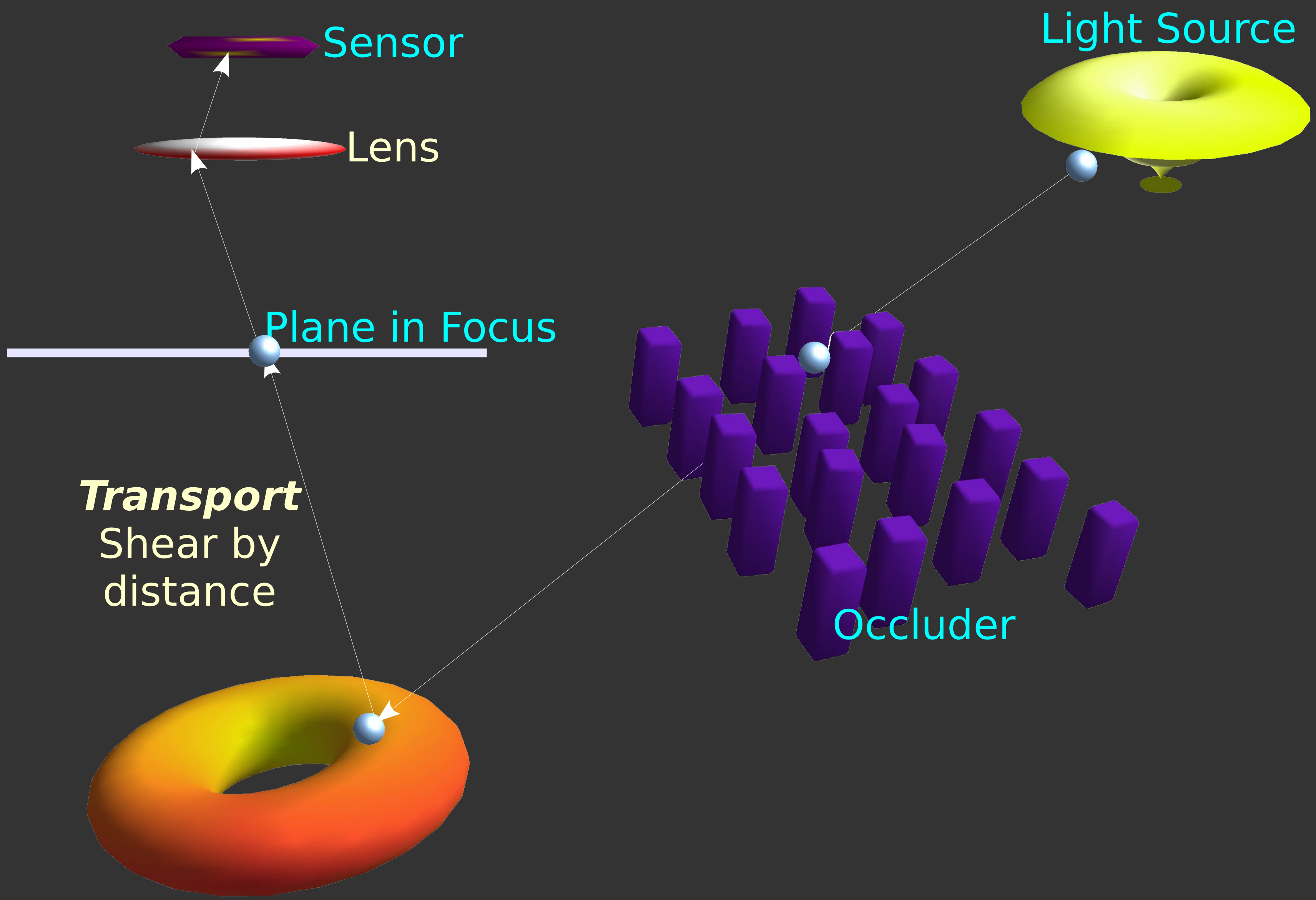
Frequency Transport



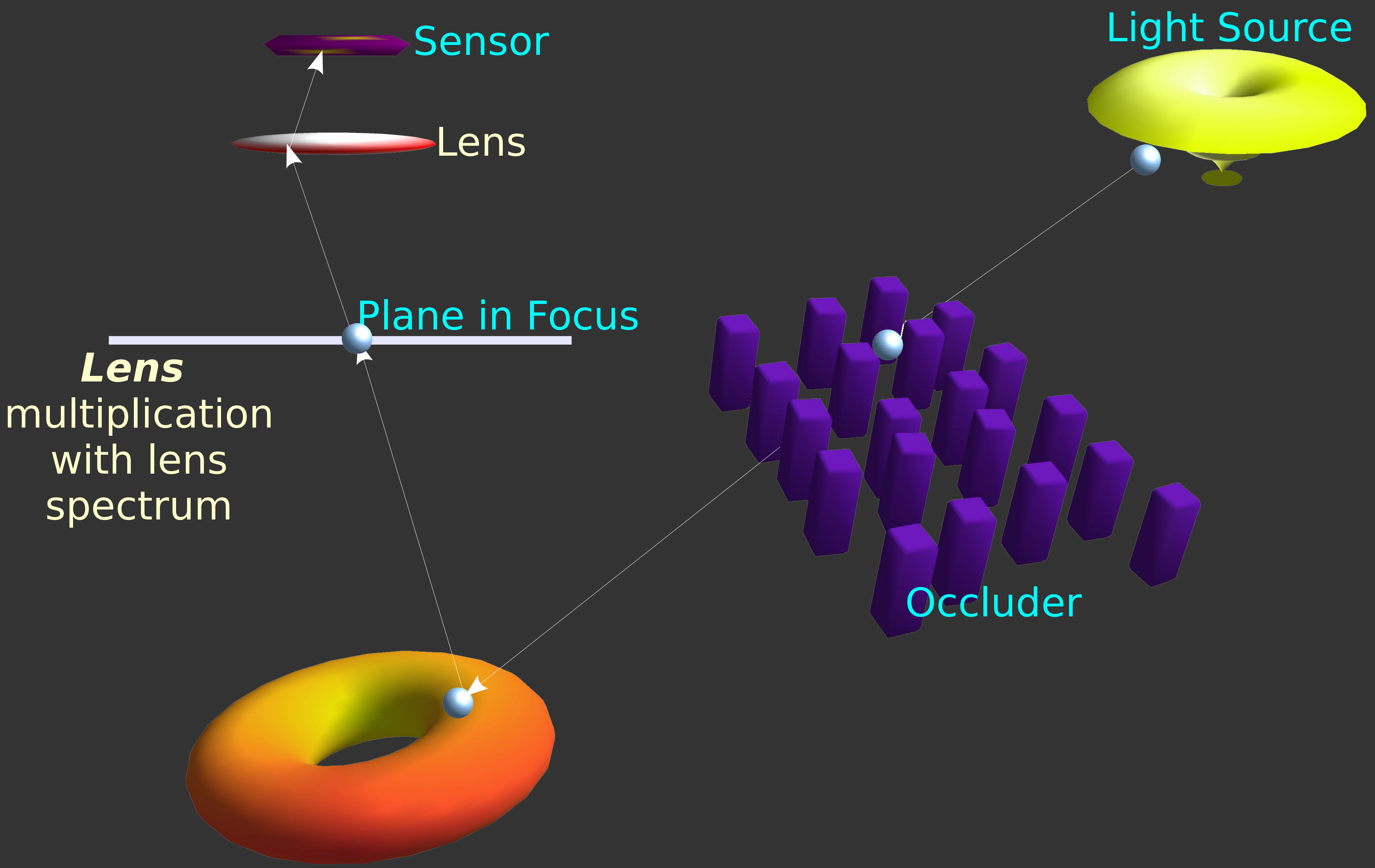
Frequency Transport



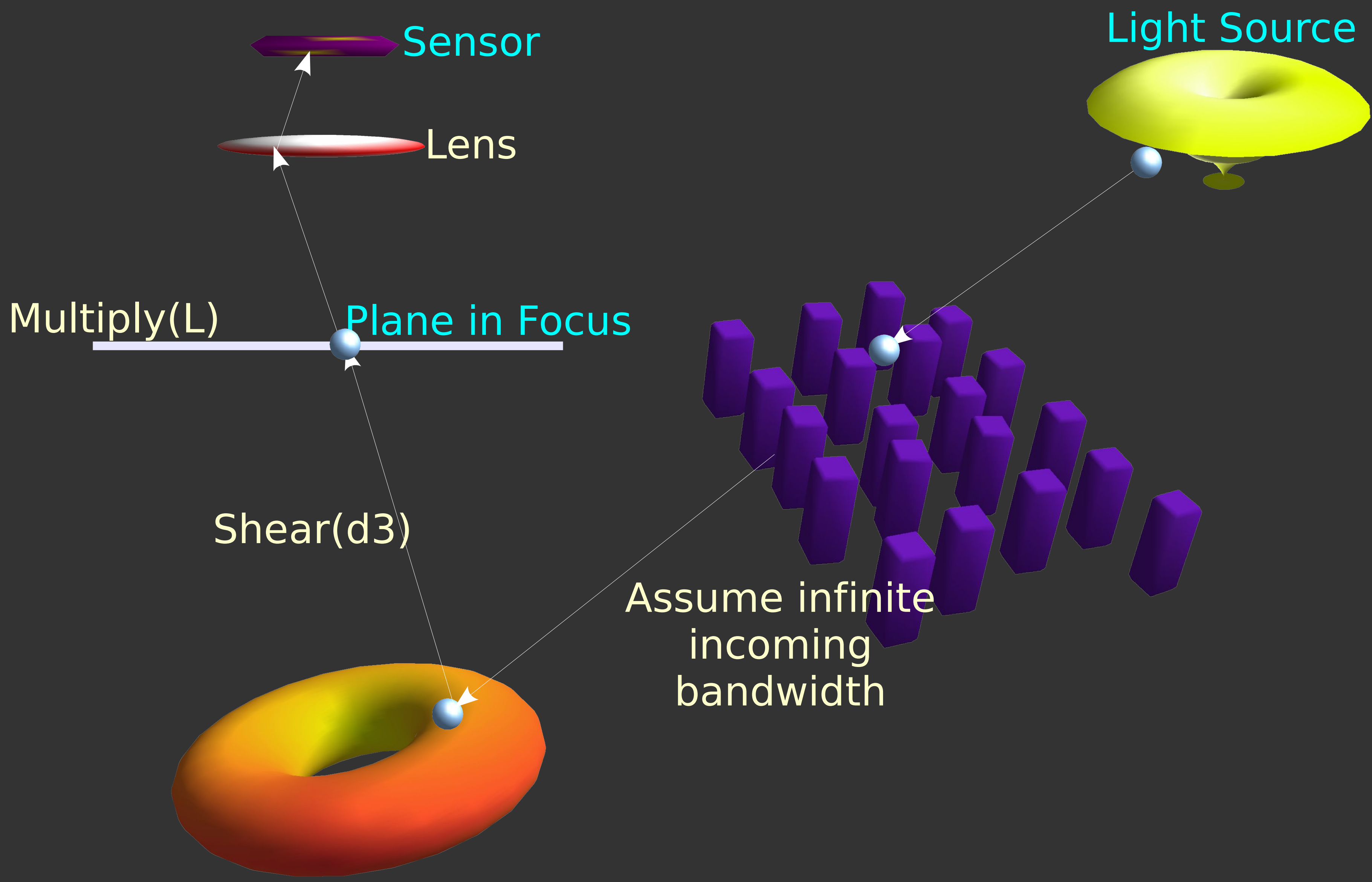
Frequency Transport



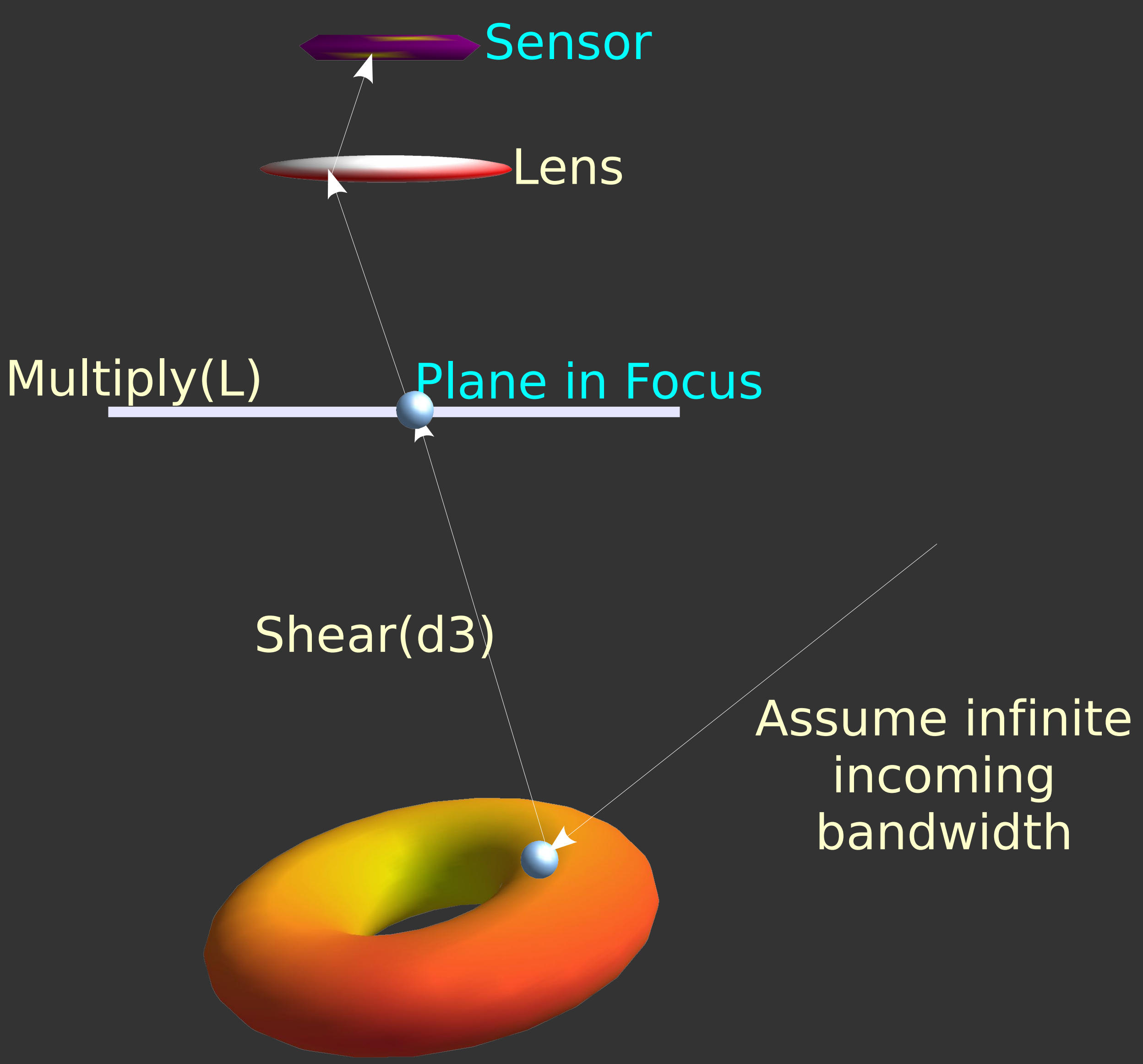
Frequency Transport



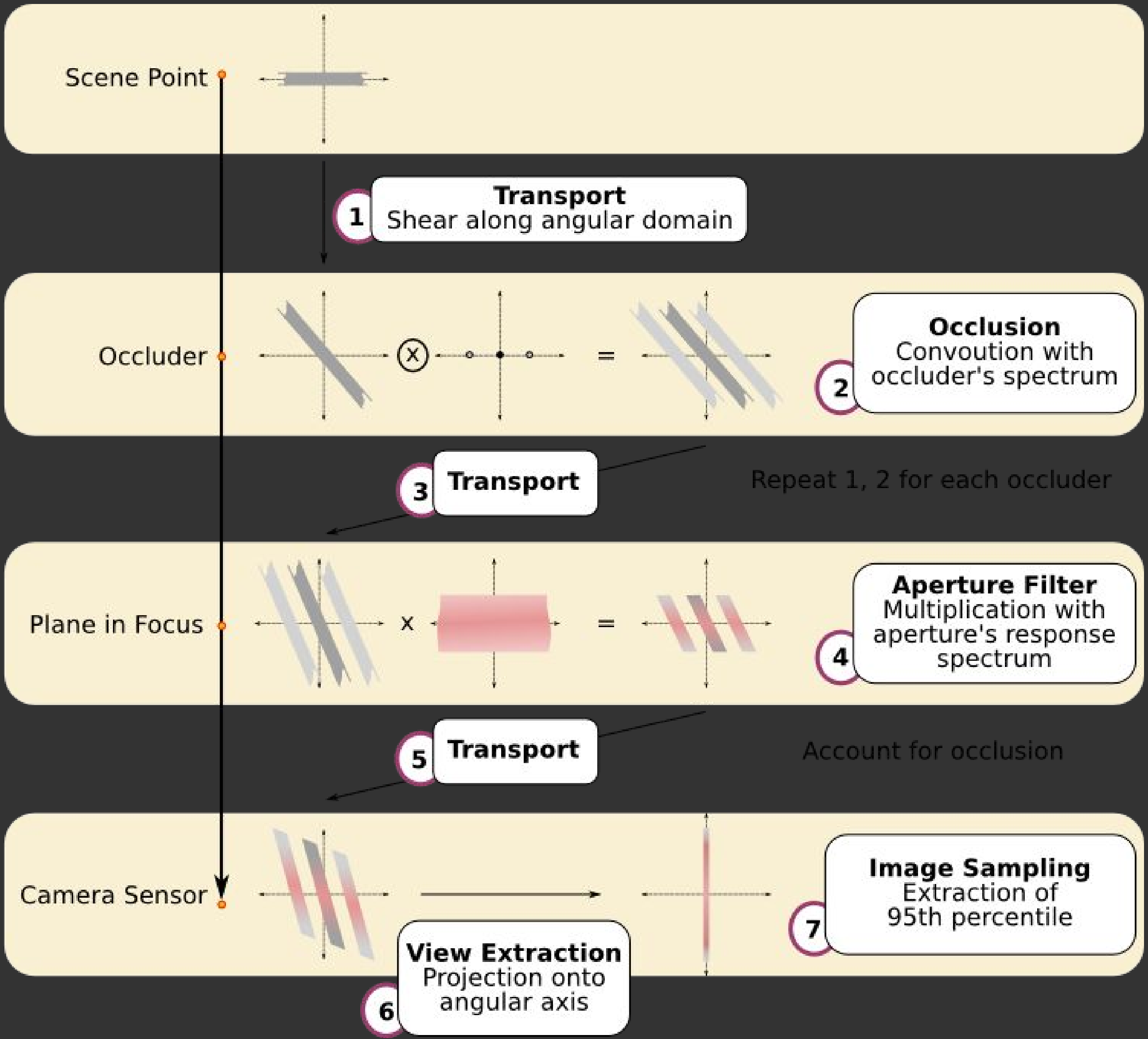
Suboptimal but conservative



Suboptimal but conservative

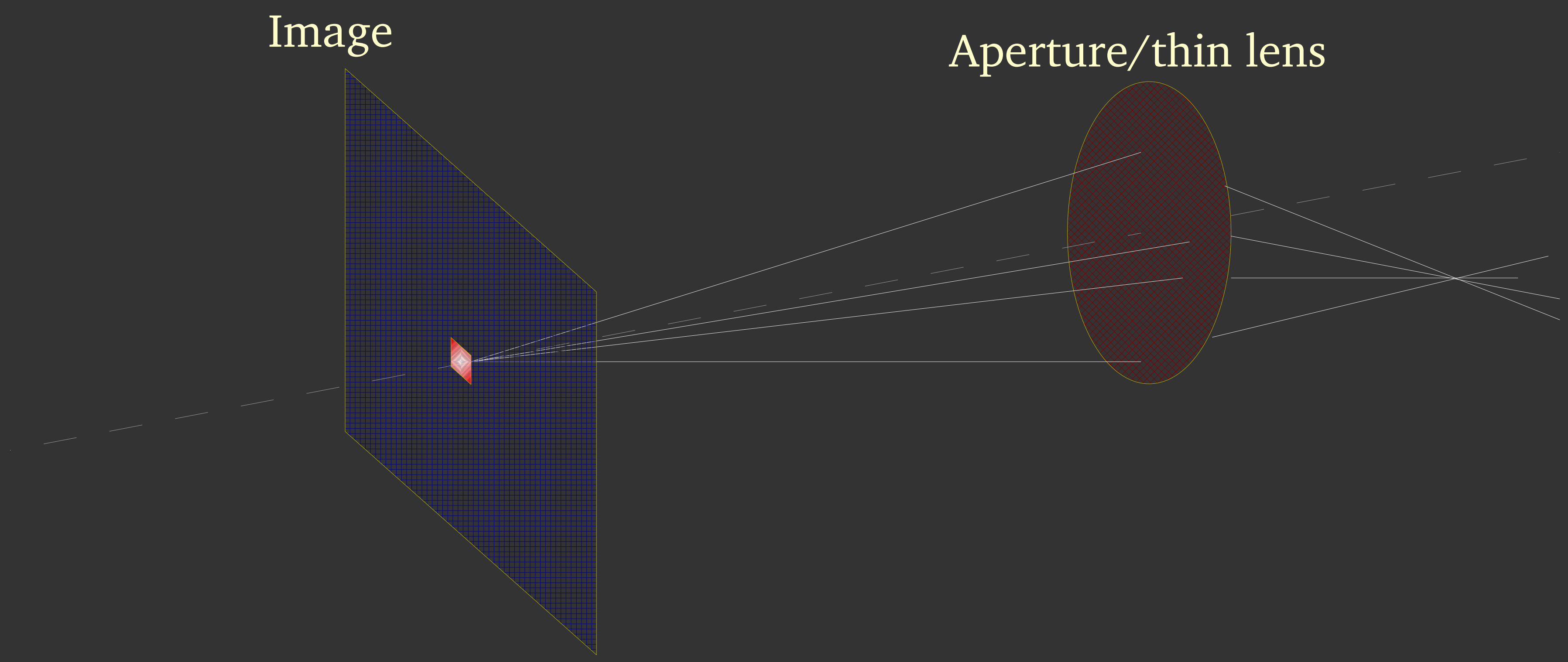


Depth of field: Fourier domain



1

Image/aperture bandwidth



Image/aperture bandwidth

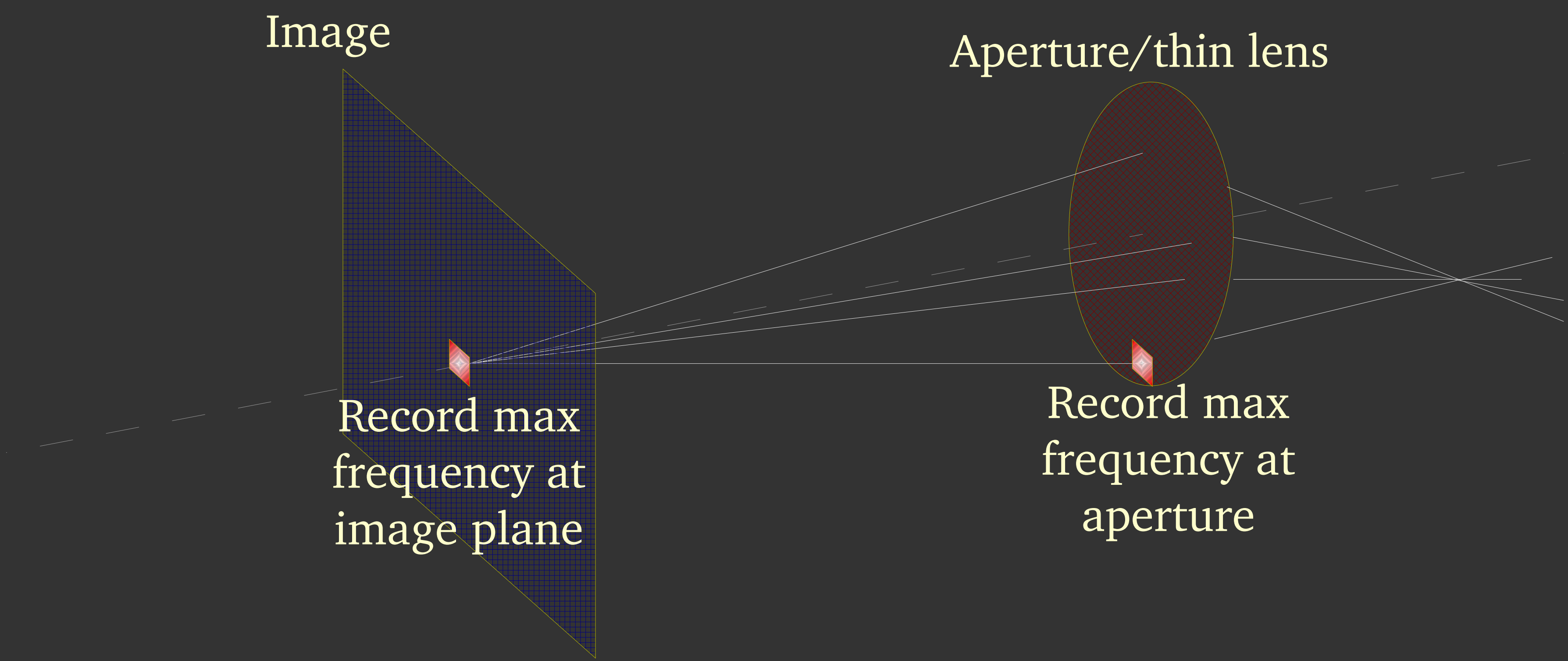


Image-space frequencies

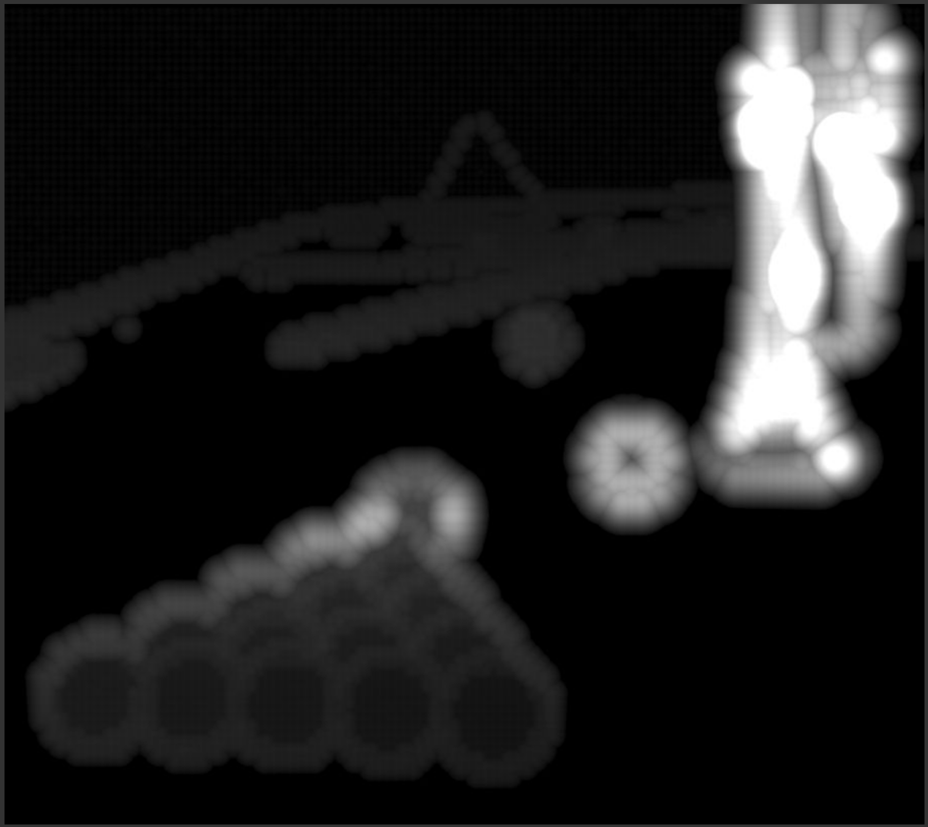


Image space sampling density

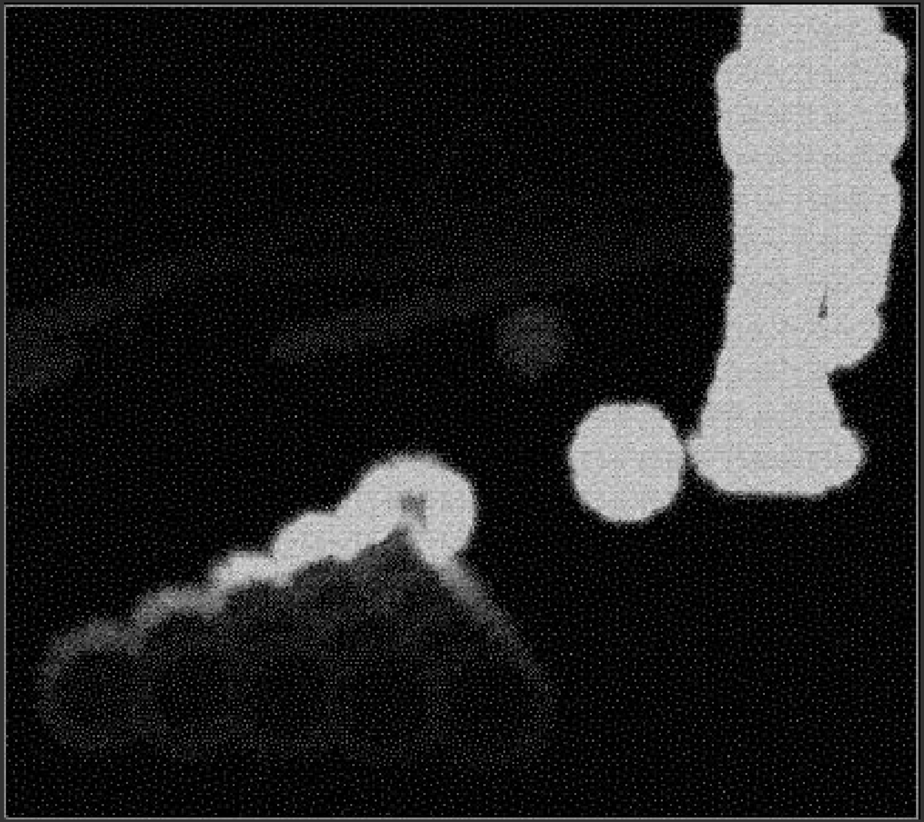
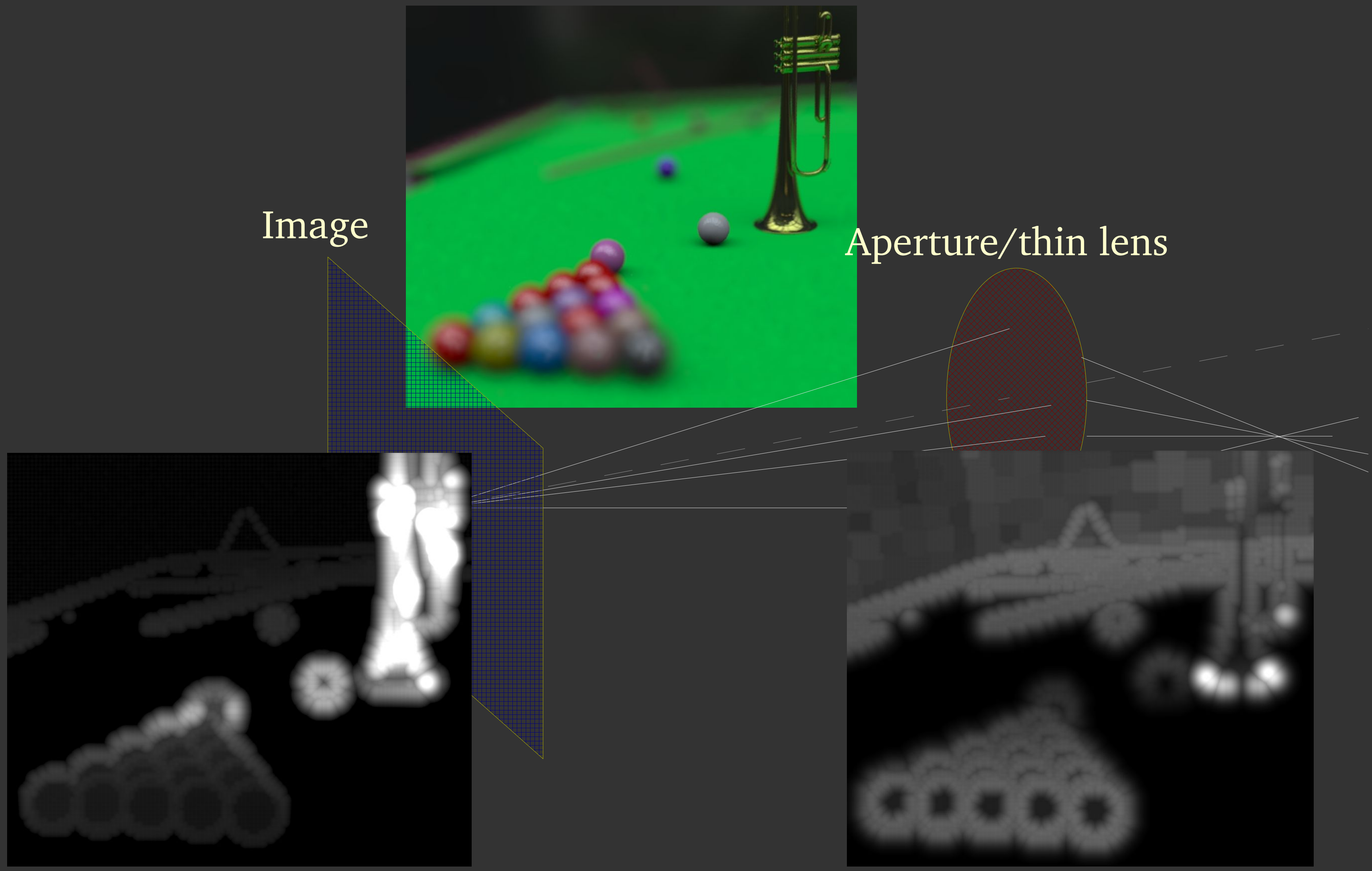


Image samples

Aperture-space bandwidth

- Expected variance in radiance estimates at each pixel
- Allocation proportional to variance

Image/aperture bandwidth

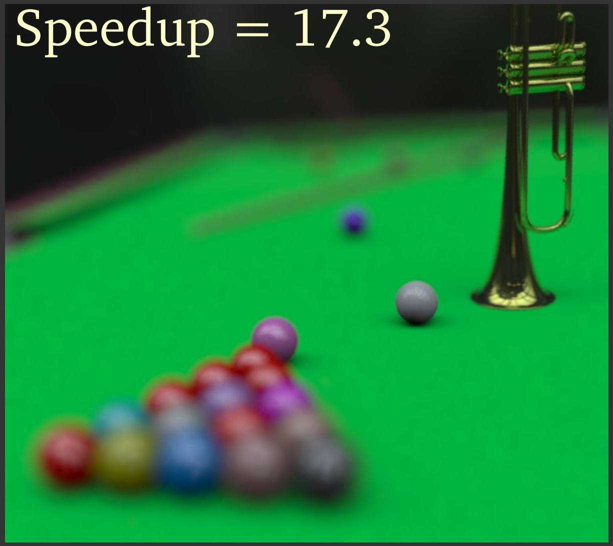


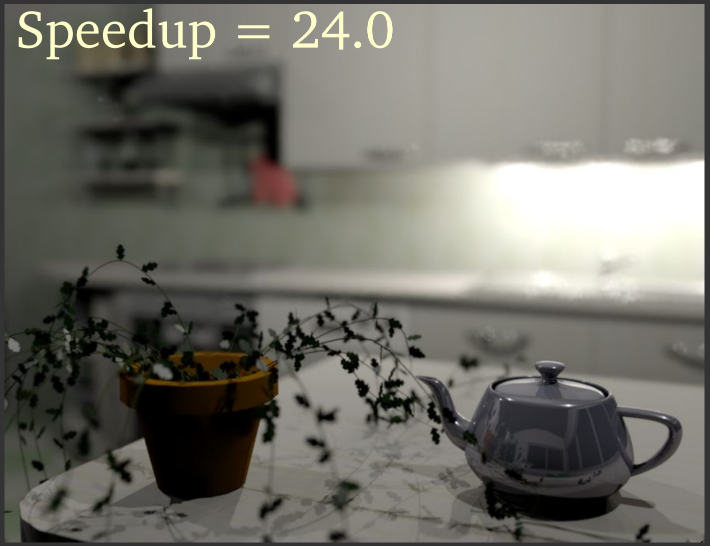
Similar cost

Without bandwidth prediction

Using bandwidth prediction

Similar quality





$$\text{Speedup} = \frac{\text{\#Primary rays using existing technique}}{\text{\#Primary rays using bandwidth prediction}} \quad \text{for similar range of noise}$$

Questions ?

[Ramamoorthi & Hanrahan 2001]

An efficient representation for irradiance environment maps.

SIGGRAPH 2001.

[Durand et al. 2005]

F.Durand, N.Holzschuch, C.Soler, F.Sillion. A frequency analysis of light transport. SIGGRAPH 2005.

Steerable Importance Sampling for Efficient Direct Distant Illumination

Kartic Subr

James Arvo

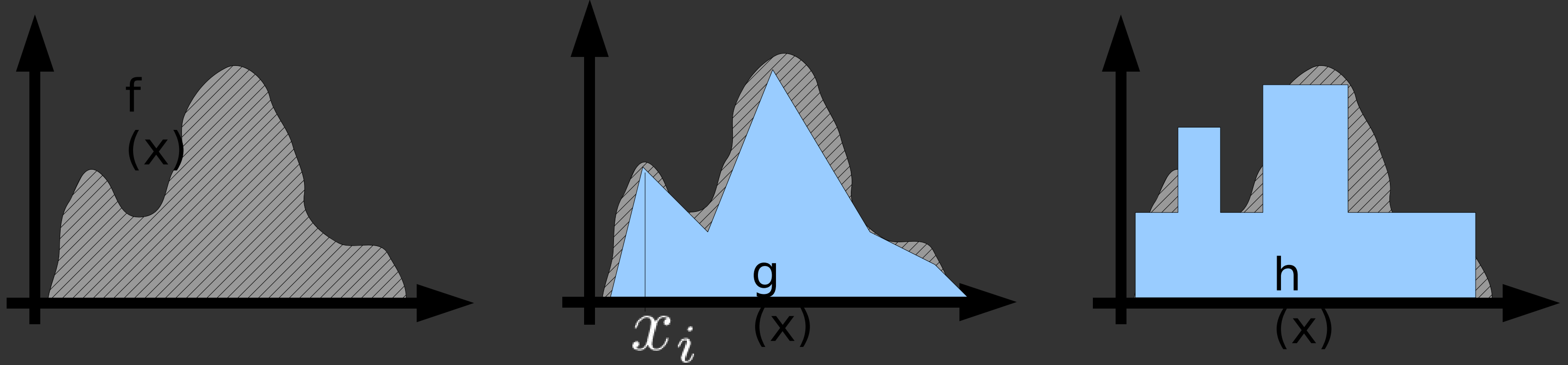
Review: Importance sampling

$$\int_{\mathcal{D}} f(x)\,\mathrm{d}x \;\approx\; \frac{1}{N}\int_{\mathcal{D}} g(x)\,\mathrm{d}x \;\sum_{i=1}^{N} \frac{f(x_i)}{g(x_i)}$$

where $x_i \;\sim\; g(x)$

Review: Importance sampling

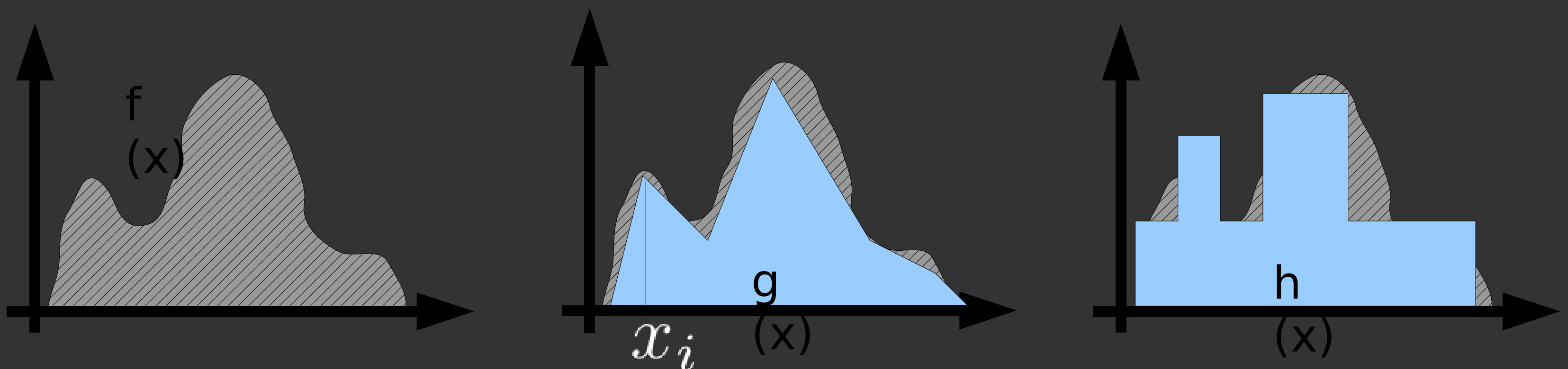
$$\int_{\mathcal{D}} f(x)\,\mathrm{d}x \;\approx\; \frac{1}{N}\int_{\mathcal{D}} g(x)\,\mathrm{d}x \sum_{i=1}^{N} \frac{f(x_i)}{g(x_i)}$$



Which is a better importance function, *g* or *h* ?

Review: Importance sampling

$$\int_{\mathcal{D}} f(x)\,\mathrm{d}x \approx \frac{1}{N}\int_{\mathcal{D}} g(x)\,\mathrm{d}x \sum_{i=1}^{N} \frac{f(x_i)}{g(x_i)}$$



Which is a better importance function, g or h ?

What if *f(x)* changes ?

Review: Steerable functions

- Transformed functions = linear combination of basis

Transformed Function

Inner Product

$$g_T(x) = < \mathbf{s}(T), \mathbf{b}(x) >$$

bases

Transformation-dependent coefficients

Direct, distant illumination

Reflected radiance along direction ω_o

$$\int_{S^2} \underbrace{V(x, \omega_i)}_{\text{Visibility}} \underbrace{\rho(\omega_o, \omega_i)}_{\text{Reflectance Function}} \underbrace{L(\omega_i)}_{\text{Incident Radiance}} \underbrace{\max(\omega_i . \mathbf{n},\ 0)}_{\text{Clamped Cosine}} \, \mathrm{d}\omega_i$$

Direct, distant illumination

Reflected radiance along direction ω_o

$$\int_{S^2} \underbrace{V(x,\omega_i)}_{\text{Visibility}} \underbrace{\rho(\omega_o,\omega_i)}_{\text{Reflectance Function}} \underbrace{L(\omega_i)}_{\text{Incident Radiance}} \underbrace{\max(\omega_i.\mathbf{n},\, 0)}_{\text{Clamped Cosine}} \, \mathrm{d}\omega_i$$

Visibility

Reflectance Function

Incident Radiance x Clamped Cosine

Importance Function

Domain Partitioning

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n})\, d\omega_i$$

Partition into Spherical Triangles

$$\left(\int + \int + \int + \ldots\right) \gamma(x, \omega_i, \omega_r, \mathbf{n})\, d\omega_i$$

Change of variables – 1

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n}) \; d\omega_i$$

Spherical to planar triangle

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n}) \; \varphi(\omega_i, \mathbf{n}_{\triangle}) \; d\omega_i$$

Planar triangle normal

Change of variables – 2

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n}) \, d\omega_i$$

Spherical to planar triangle

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n}) \, \varphi(\omega_i, \mathbf{n}_\triangle) \, d\omega_i$$

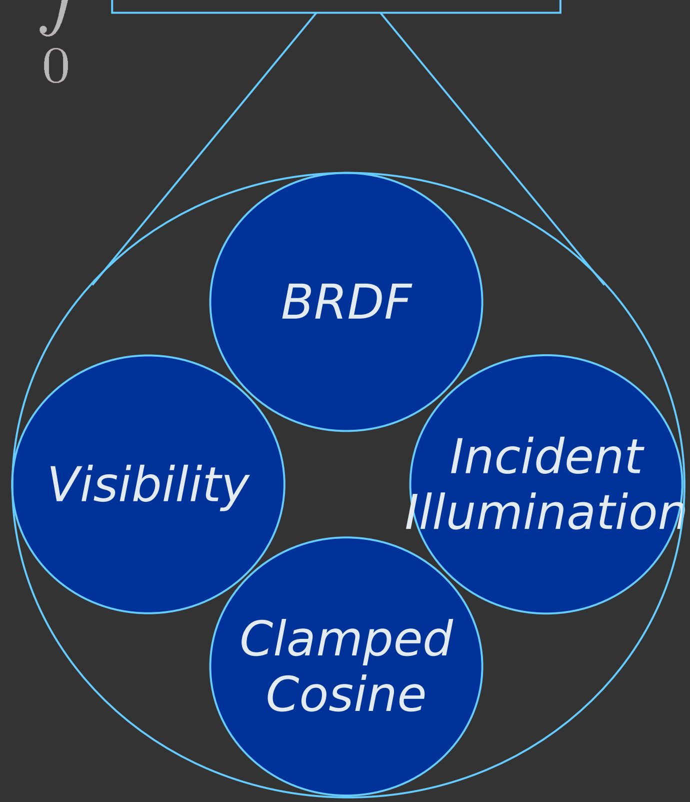
Unit square to triangle parameterization

$$\int_0^1 \int_0^1 \gamma(x, \omega_i, \omega_r, \mathbf{n}) \, \varphi(p_0, p_1, \mathbf{n}_\triangle) \, |J(p_0, p_1)| \, dp_0 \, dp_1$$

Jacobian of Parameterization

Novel parameterization

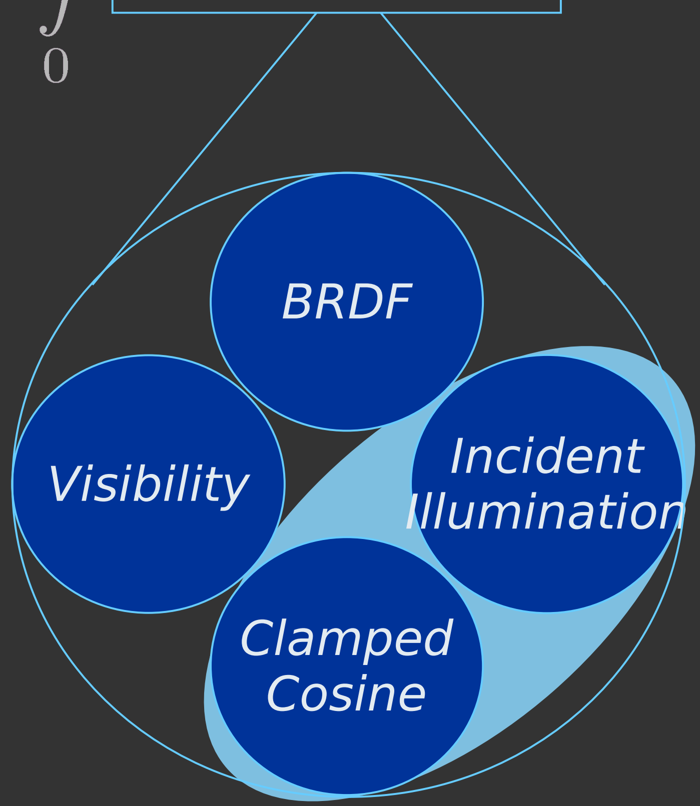
$$\int_0^1 \int_0^1 \gamma(x, \omega_i, \omega_r, \mathbf{n})\, \varphi(p_0, p_1, \mathbf{n}_\triangle)\, |J(p_0, p_1)|\, dp_0\, dp_1$$



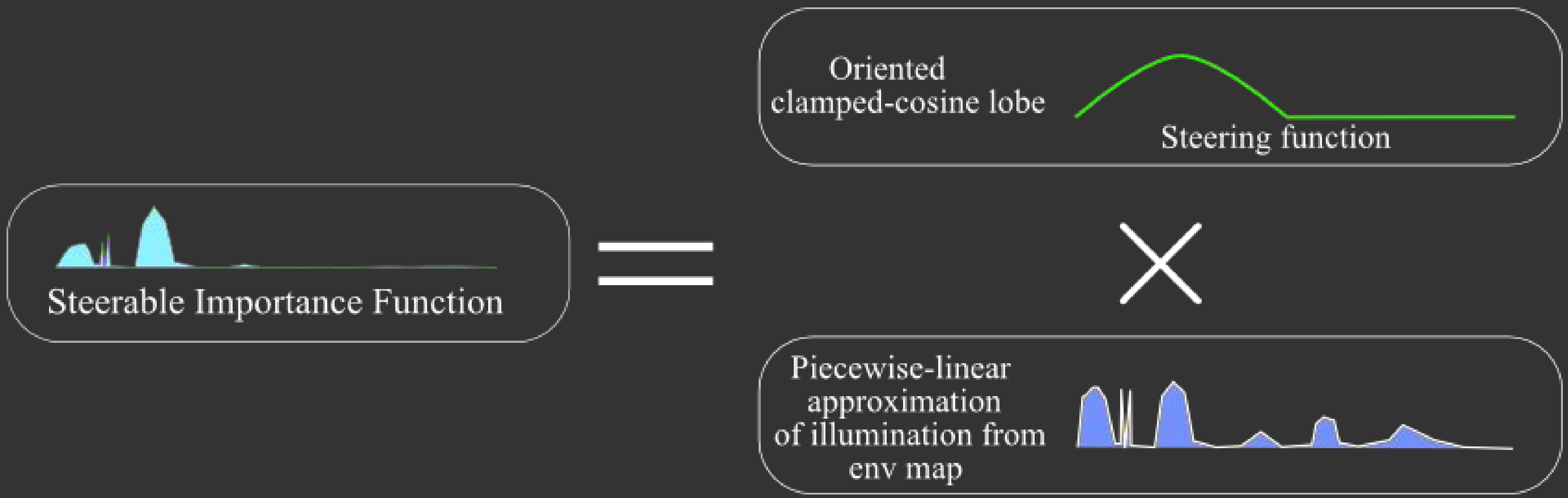
Novel parameterization

$$\int_0^1 \int_0^1 \boxed{\gamma(x, \omega_i, \omega_r, \mathbf{n})}\, \varphi(p_0, p_1, \mathbf{n}_\triangle)\, \boxed{|J(p_0, p_1)|}\, dp_0\, dp_1$$

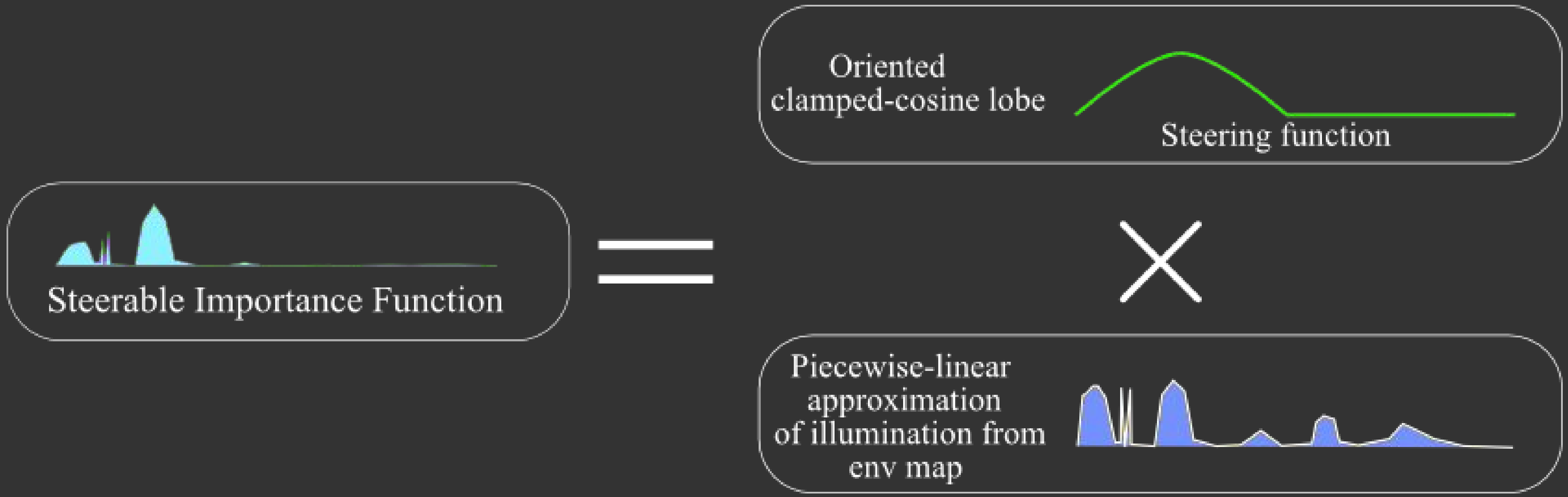
Derive parameterization so that
Jacobian $\approx$ Illumination * Clamped Cosine



Steerable importance function



Steerable importance function



Is this steerable?

Steerable importance function

$$g_{\mathbf{n}}(\mathbf{u}) = L(\mathbf{u}) \max(\mathbf{n}.\mathbf{u}, 0)$$

Steerable importance function

$$g_{\mathbf{n}}(\mathbf{u}) = L(\mathbf{u})\ \max(\mathbf{n}.\mathbf{u}, 0)$$

Represent using SH bases
8 coefficients – good approximation
[Ramamoorthi & Hanrahan]

$$<\mathbf{a}(\mathbf{n}), \mathbf{Y}(\mathbf{u})>$$

Rotated coefficients

Spherical Harmonic bases

Steerable importance function

$$g_n(\mathbf{u}) = L(\mathbf{u}) \max(\mathbf{n}.\mathbf{u}, 0)$$

$$\langle \mathbf{a}(\mathbf{n}), \mathbf{Y}(\mathbf{u}) \rangle$$

rewrite

$$g_n(\mathbf{u}) = \langle \mathbf{a}(\mathbf{n}), L(\mathbf{u})\, \mathbf{Y}(\mathbf{u}) \rangle$$

Steerable importance function

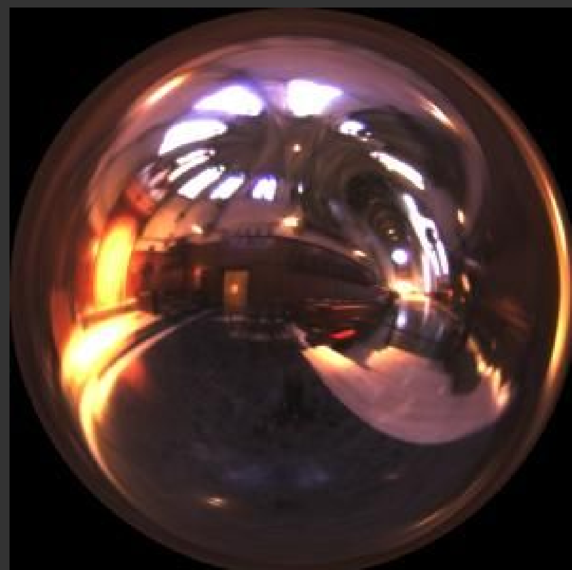
$$g_n(\mathbf{u}) = L(\mathbf{u})\,\max(\mathbf{n}.\mathbf{u}, 0)$$

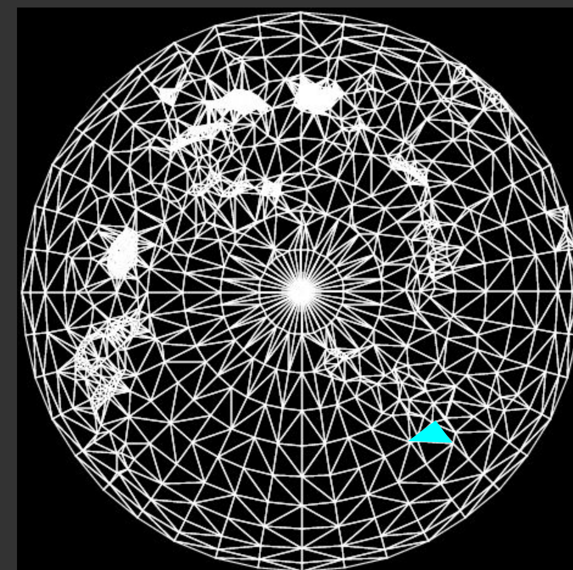
$$\langle \mathbf{a}(\mathbf{n}), \mathbf{Y}(\mathbf{u}) \rangle$$

$$g_n(\mathbf{u}) = \langle \mathbf{a}(\mathbf{n}), L(\mathbf{u})\,\mathbf{Y}(\mathbf{u}) \rangle$$

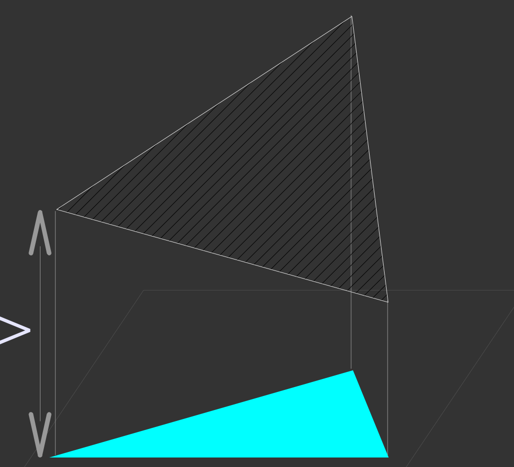
Precomputed

Steerable importance function

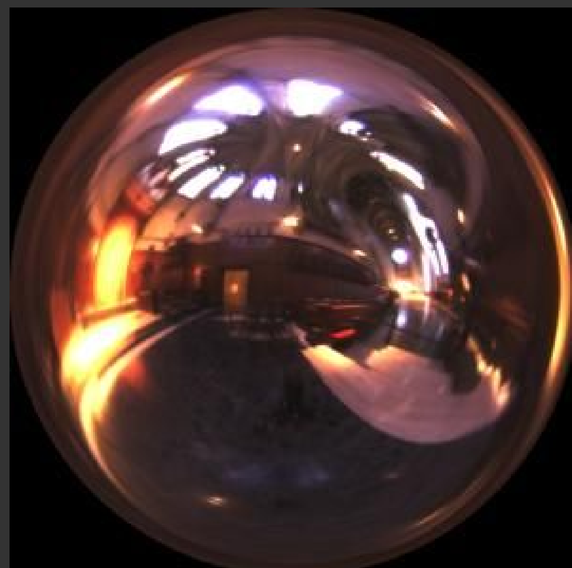


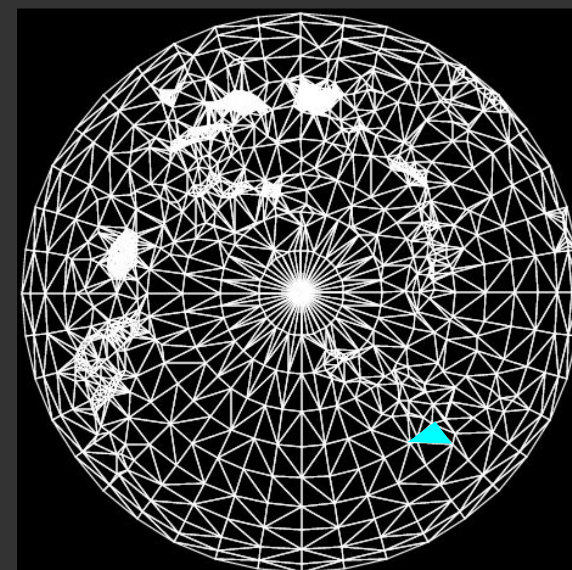


$$\langle \mathbf{a}(\mathbf{n}), L(\mathbf{u})\ \mathbf{Y}(\mathbf{u}) \rangle$$



Steerable importance function

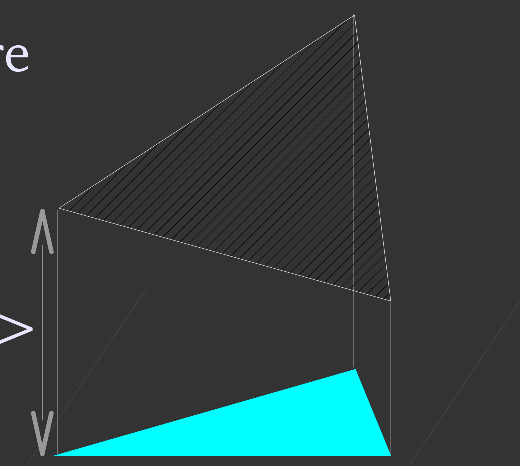




Precompute and store
(per vertex)

Function of normal

<**a(n)**, L(u) **Y(u)**>



Drawing samples

- Triangle selection

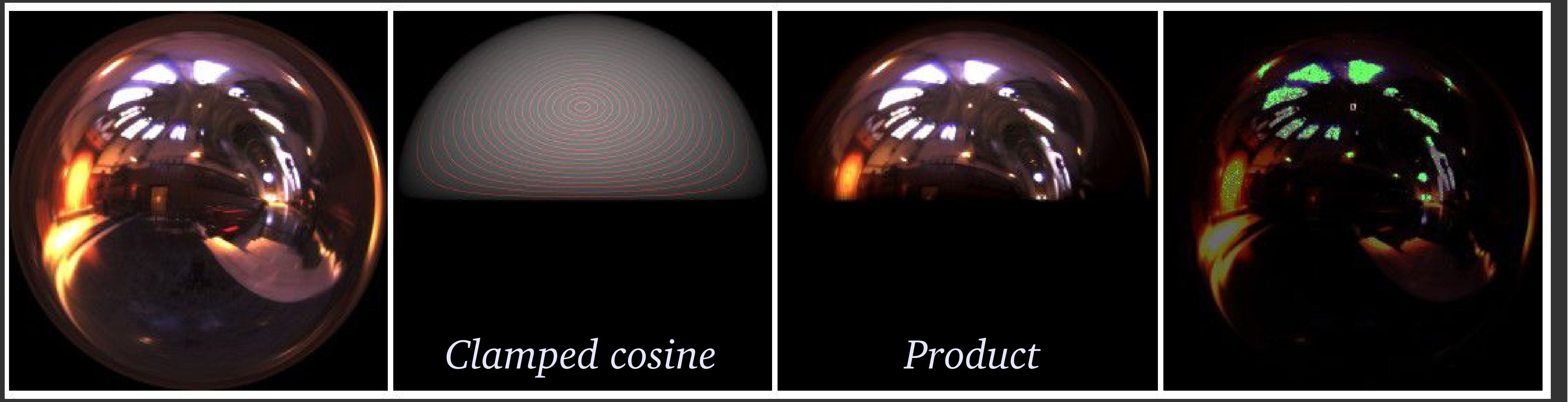
- Stratified sampling of selected triangle

Drawing samples

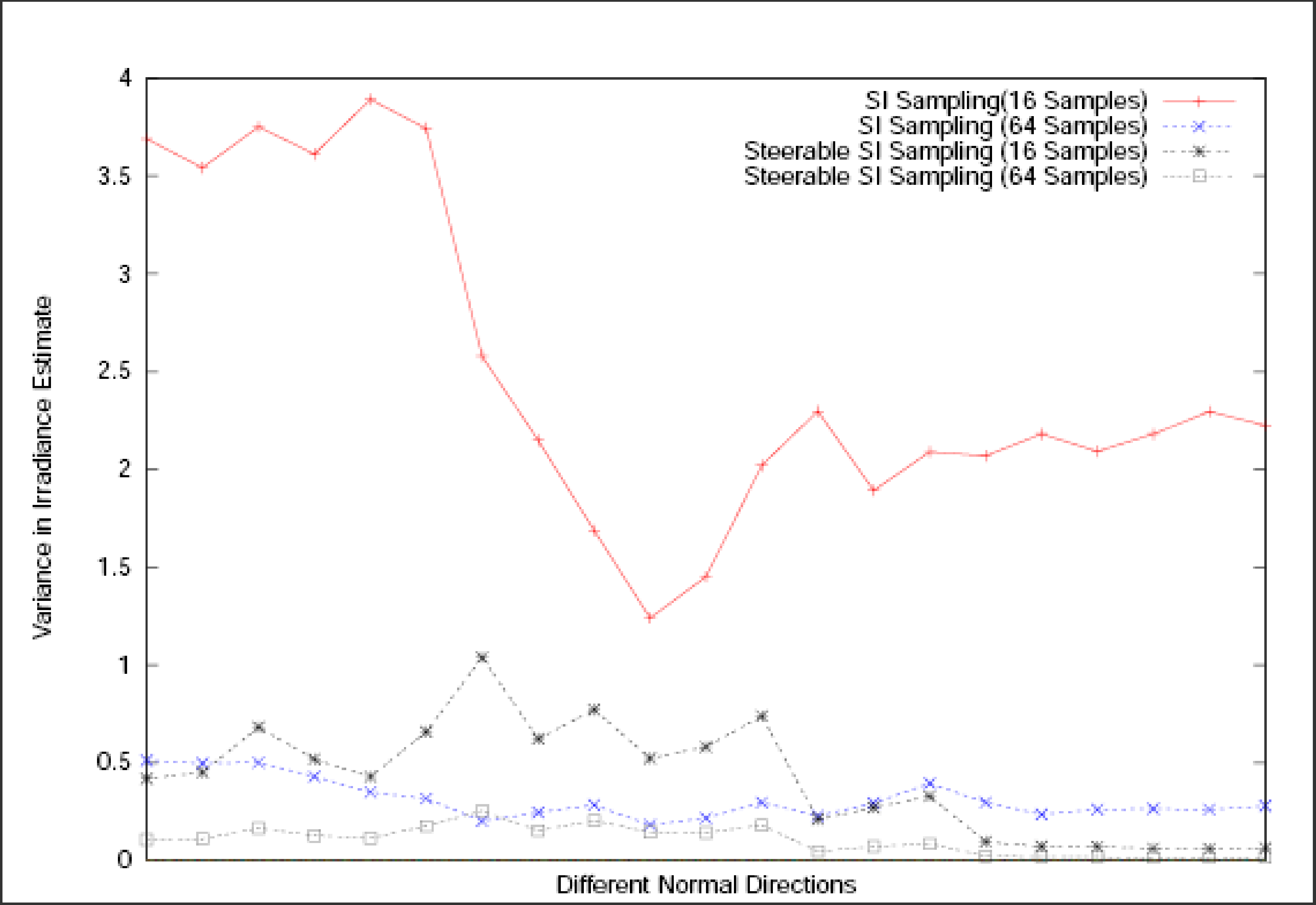
- Triangle selection
  - proportional to function integral within triangle
  - O(log *N*) cost (N triangles)

- Stratified sampling of selected triangle
  - according to linear function
  - O(1) cost

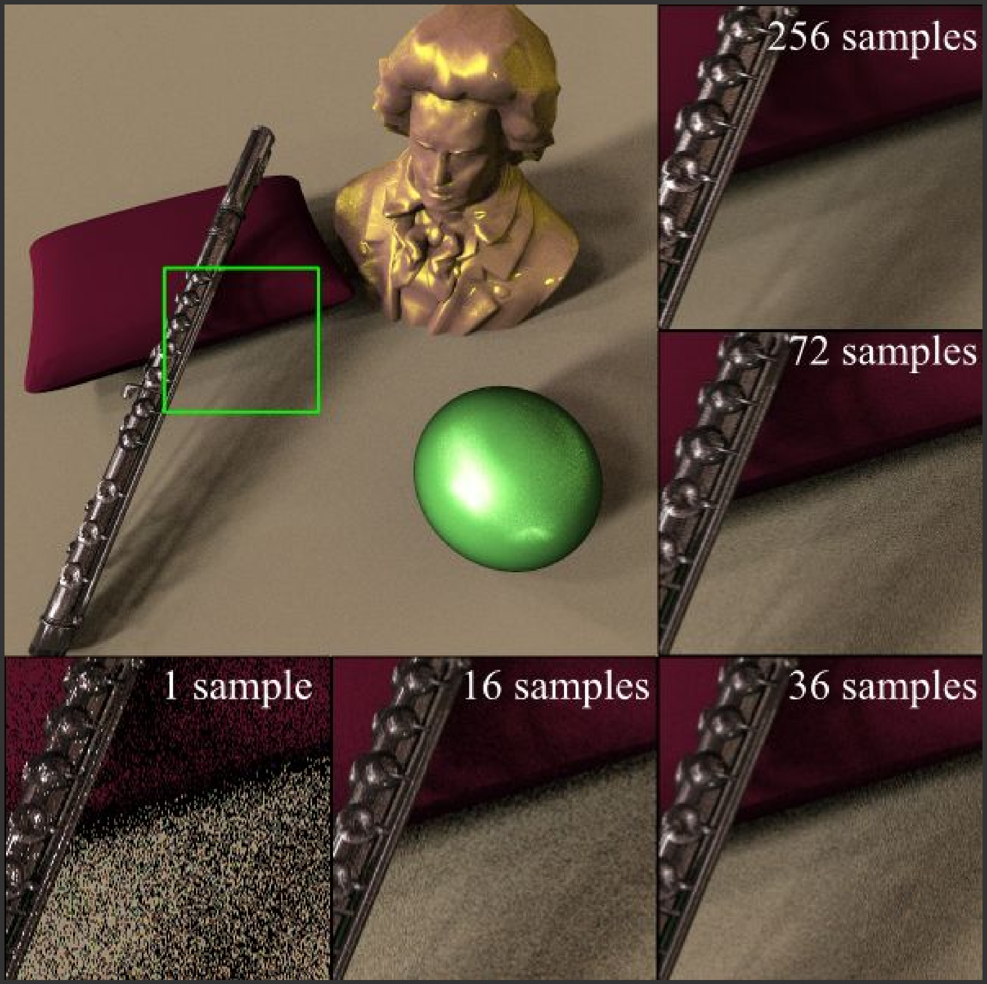
Results

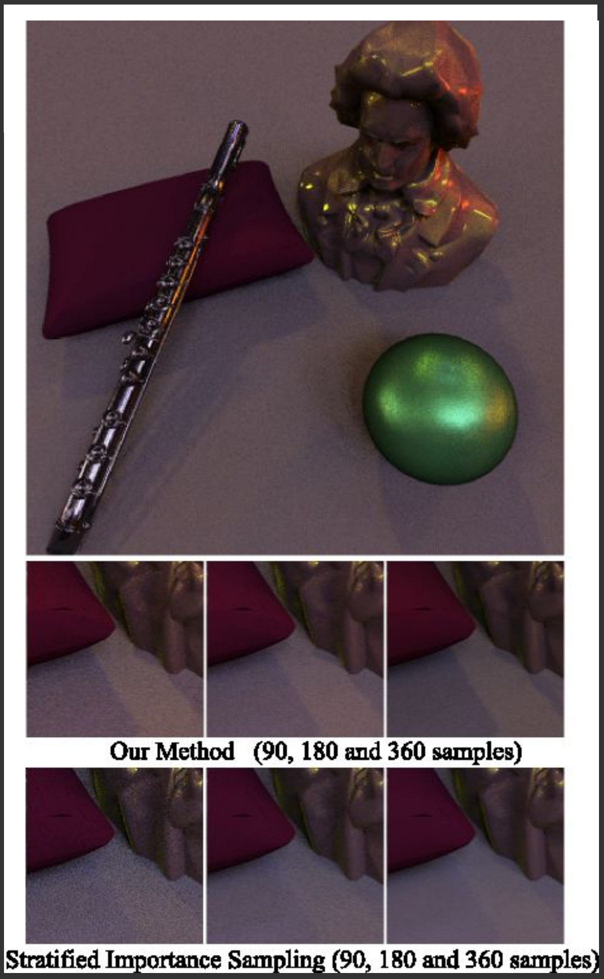

Environment map
Samples (green)
Clamped cosine
Product
Input
Output

Results: Reduced variance



Results: Images generated





Questions ?

[Ramamoorthi & Hanrahan]

An efficient representation for irradiance environment maps.

SIGGRAPH 2001.

[Teo]

Theory and applications of steerable functions.

PhD thesis, 1998.

[W. Freeman]

Steerable filters and the local analysis of image structure.

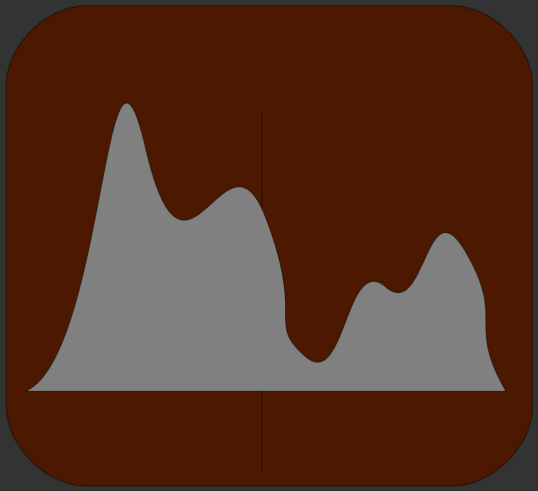
PhD thesis, 1992.

Assessing Monte Carlo Estimators: Applications in Image Synthesis

Kartic Subr James Arvo

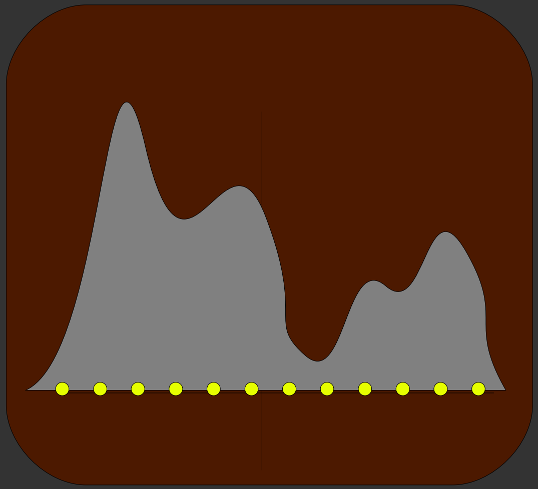
Review: Estimator

- Simple example – Estimating the Mean



Review: Estimator

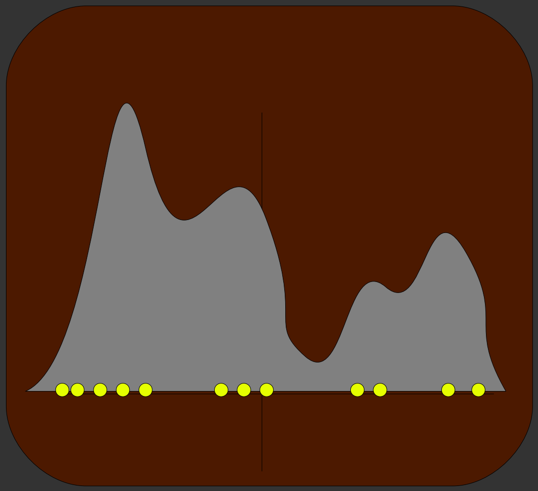
- Simple example – MC Estimator for the Mean



- Sample domain randomly (unif.)
- Average function-values at sample locations

Review: Estimator

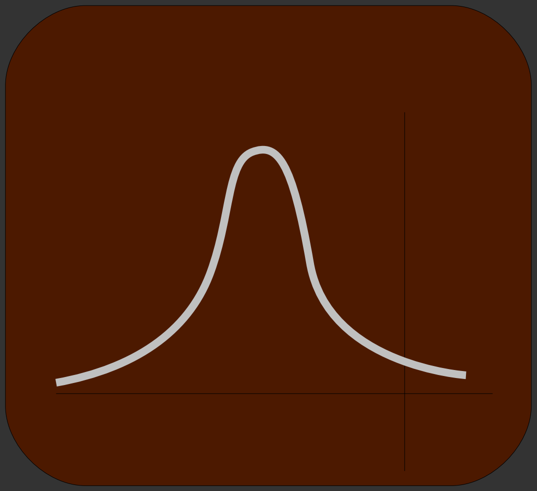
- Simple example – MC Estimator for the Mean



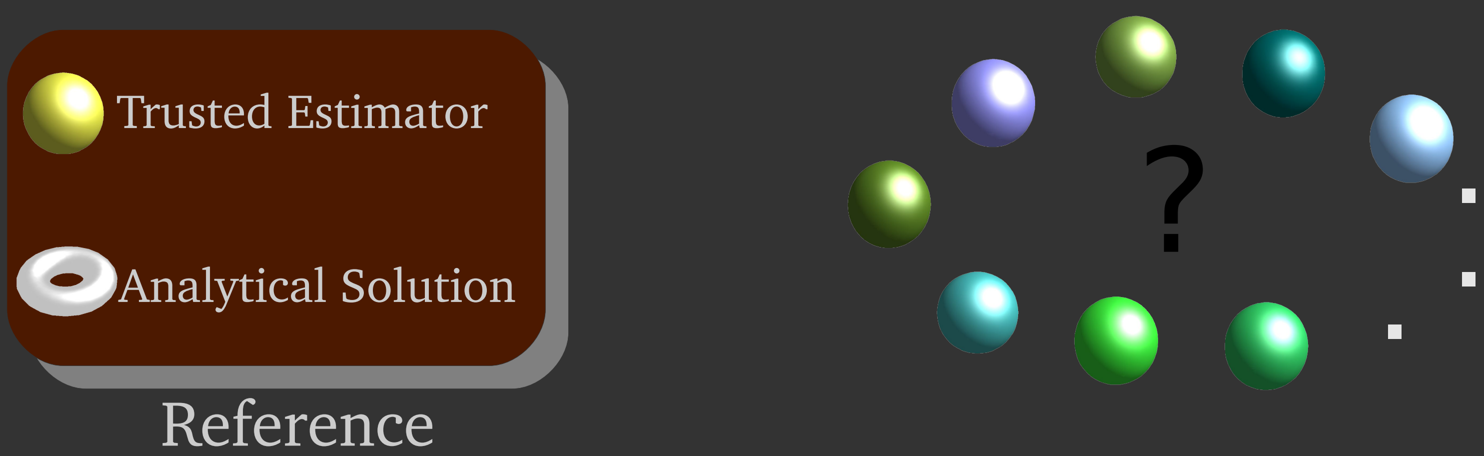
- Sample domain non-uniformly
- Average weighted function-values at sample locations
- 'Compensate' for sampling

Review: Estimator

- Repeat process
- Obtain several estimates
- Histogram of Estimates



Assessing Estimators



Assessing Estimators

Typically compare 1st and/or 2nd order statistics

i.e. Mean and Variance

Trusted Estimator

Analytical Solution

Reference

Mean( ) > Mean( )

Var( ) > Var( )

?

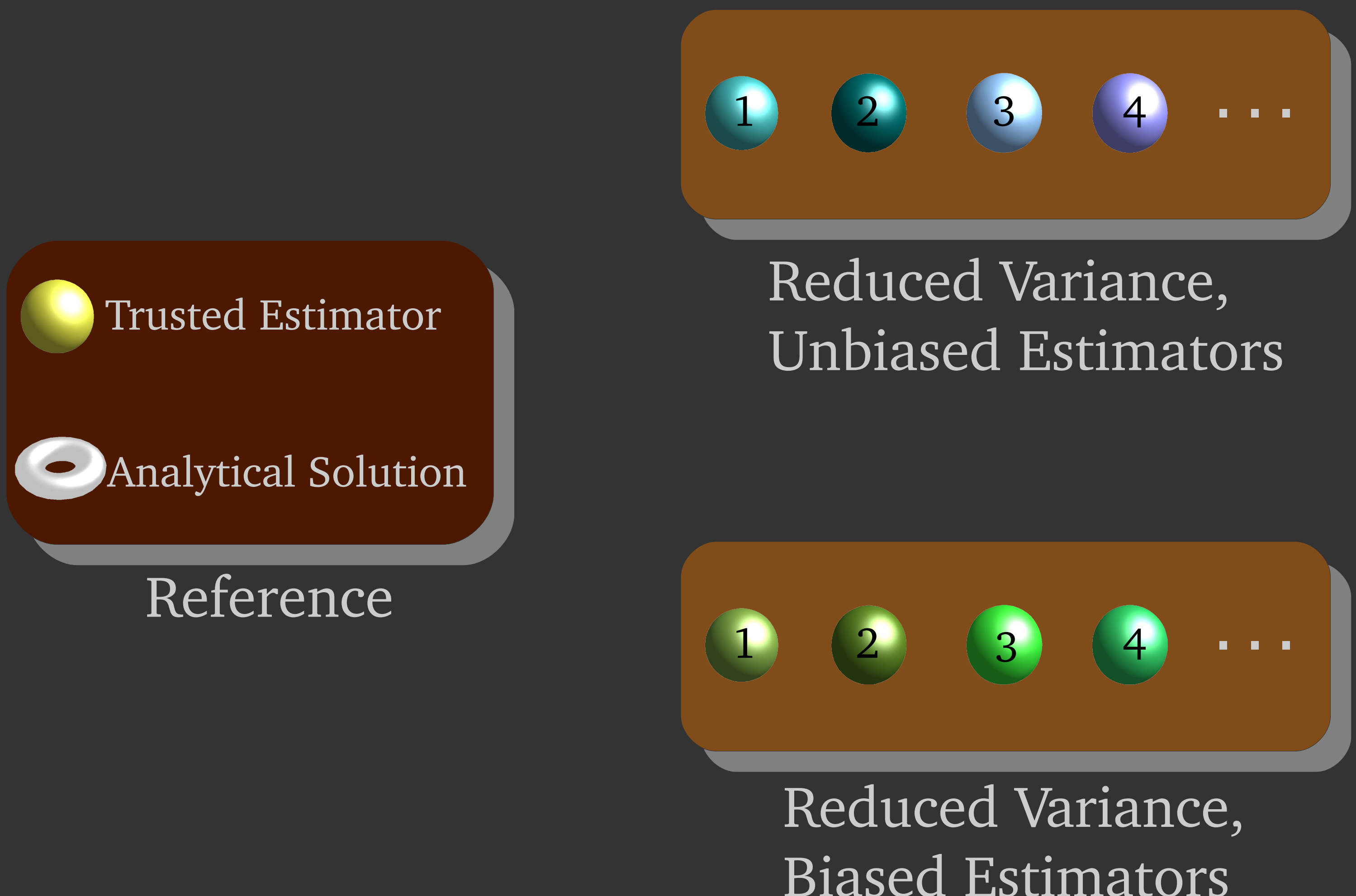
Assessing Estimators- Image Synthesis

- Cost
  - time
  - number of samples
- Mean
  - difference images
  - inspecting convergence plots
- Variance
  - inspecting image noise

Assessing Estimators- Image Synthesis

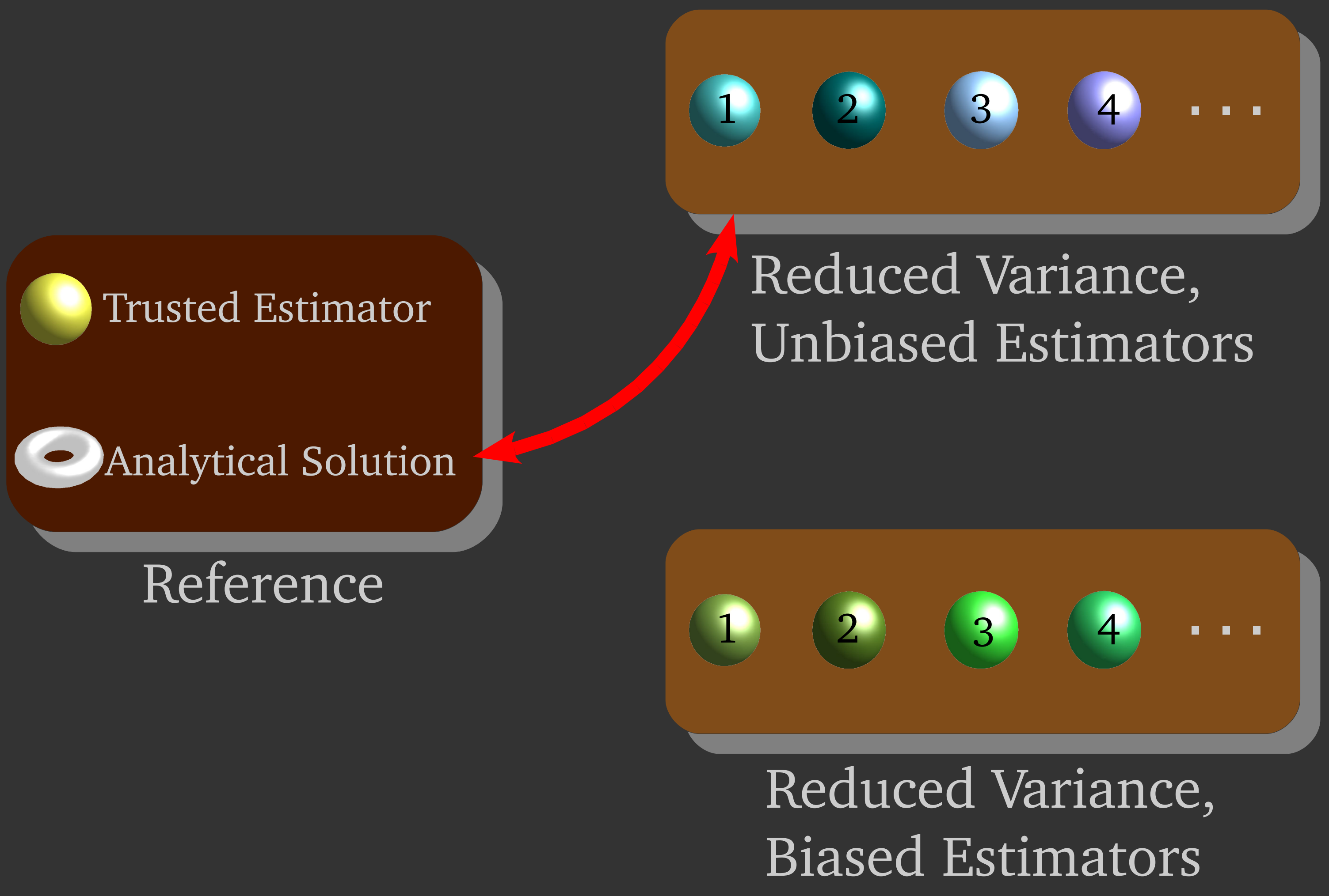
- Drawbacks (current techniques)
  - subjective
  - weakly quantitative
  - comparing variance plots- large number of estimates
  - difficult, often impossible, to automate

Typical Classes of MC Estimators in Image Synthesis



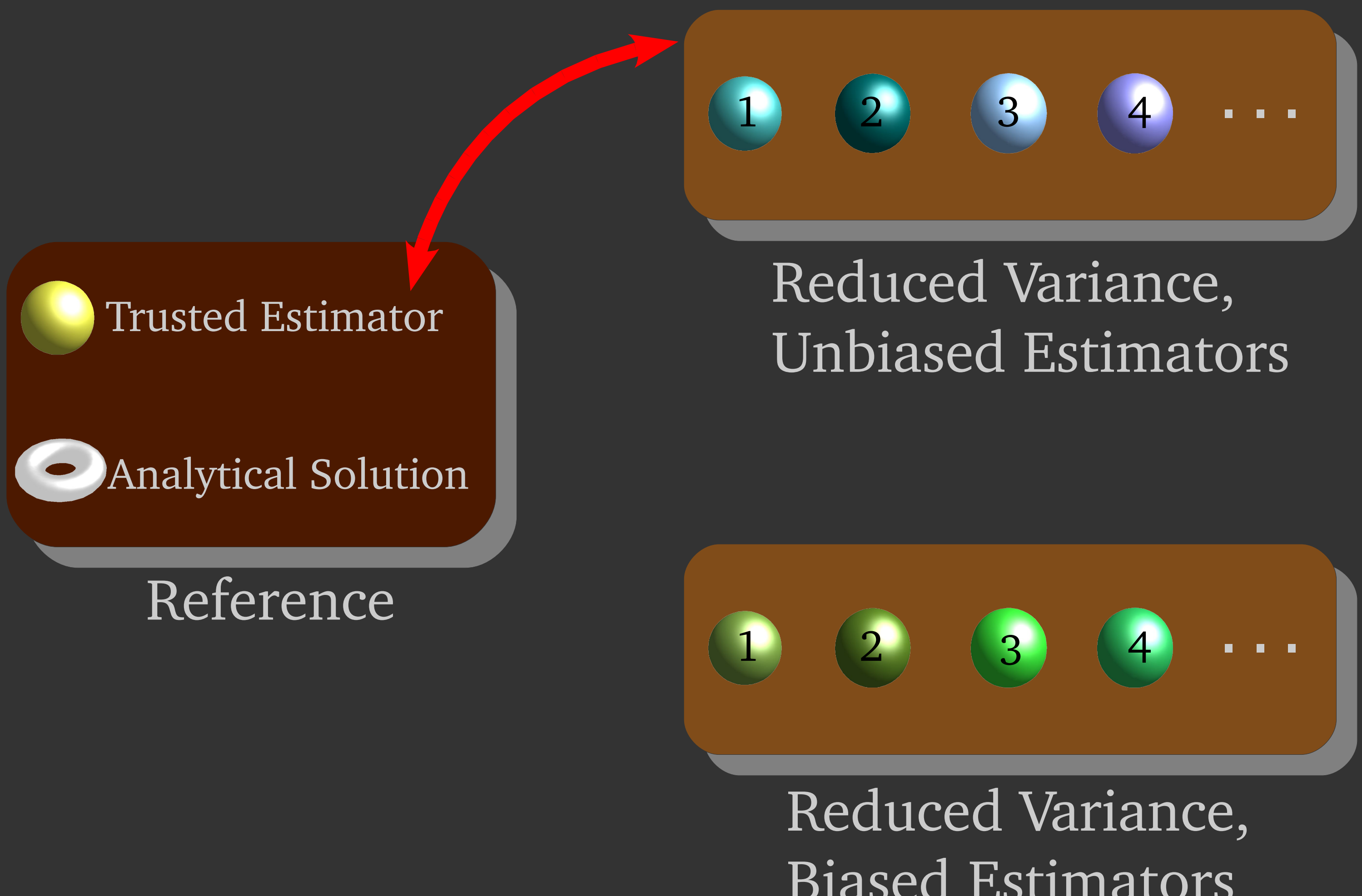
Verifying Absence of Bias

1. Estimator vs Analytical Solution



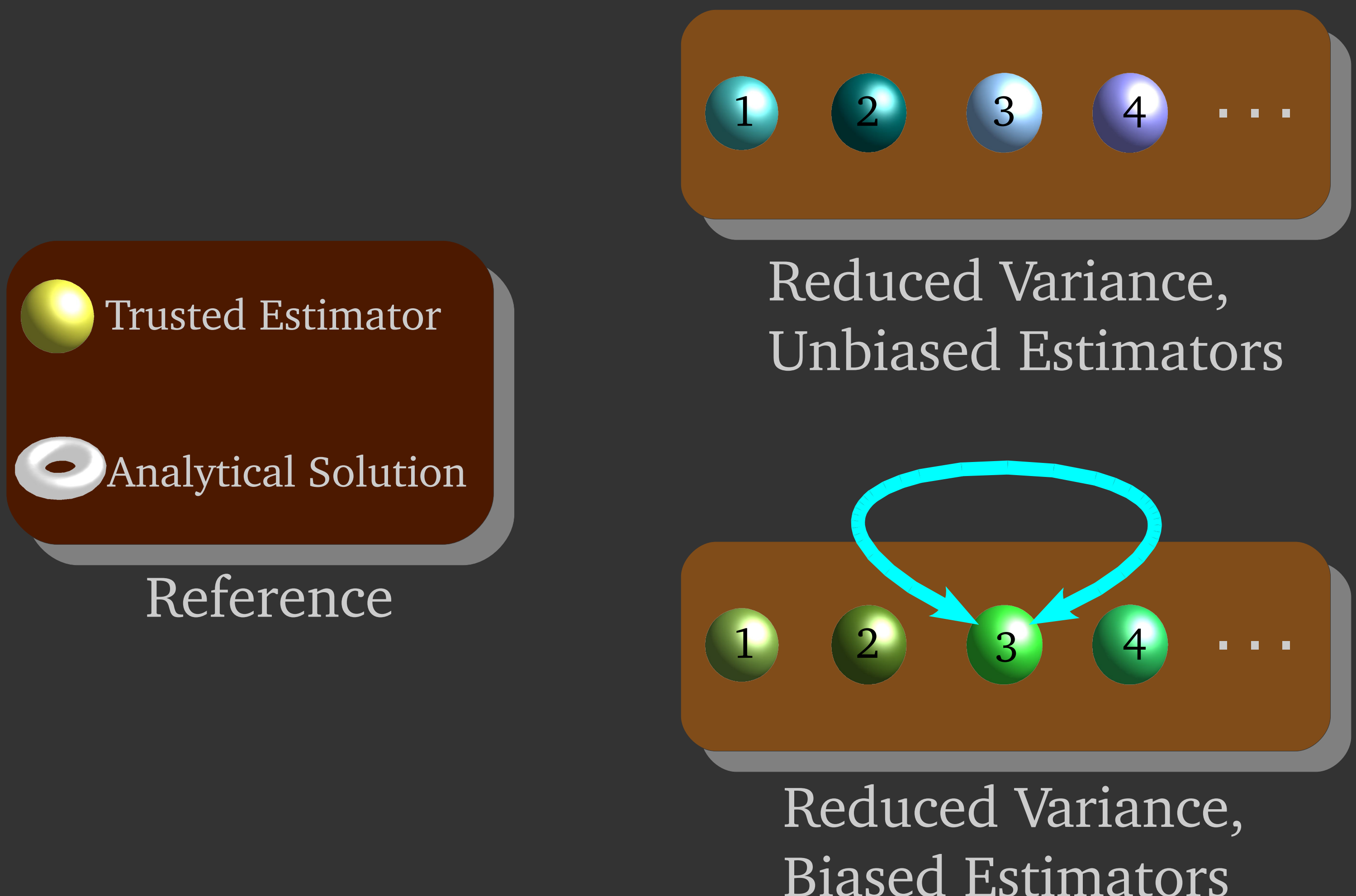
Verifying Absence of Bias

2. Estimator vs Trusted Estimator



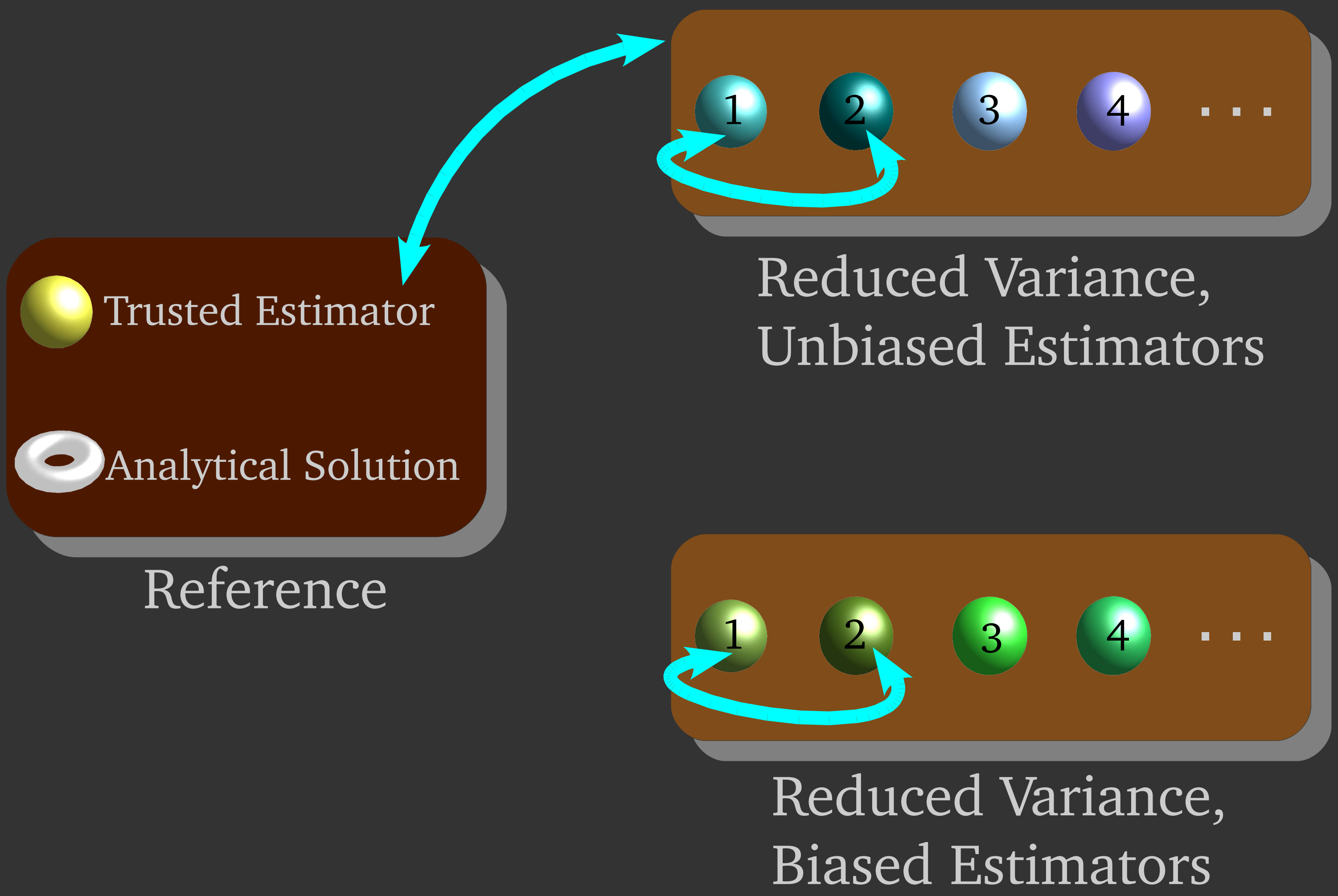
Verify Variance Acceptability

3. Verify variance acceptibility- Estimator vs Constant

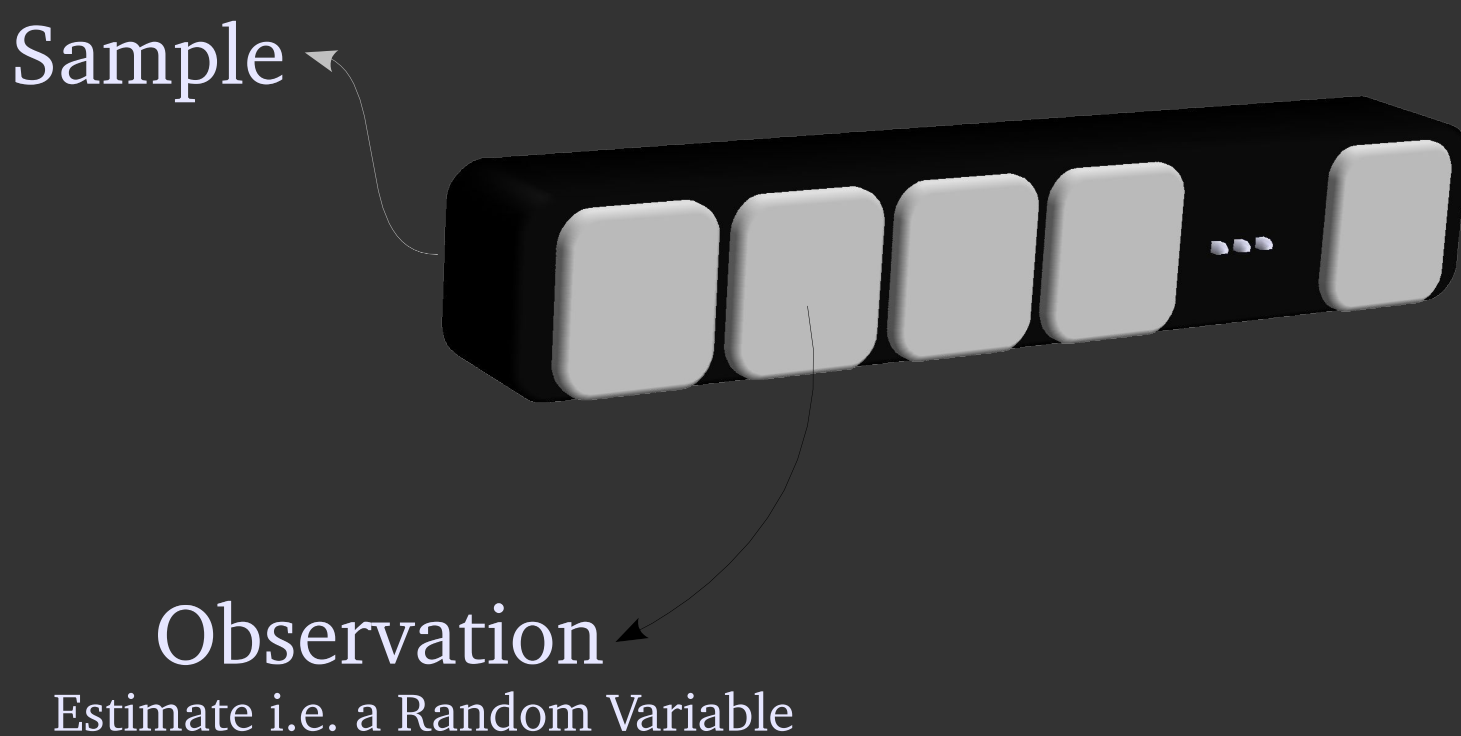


Verify Variance Reduction or Compare Variances

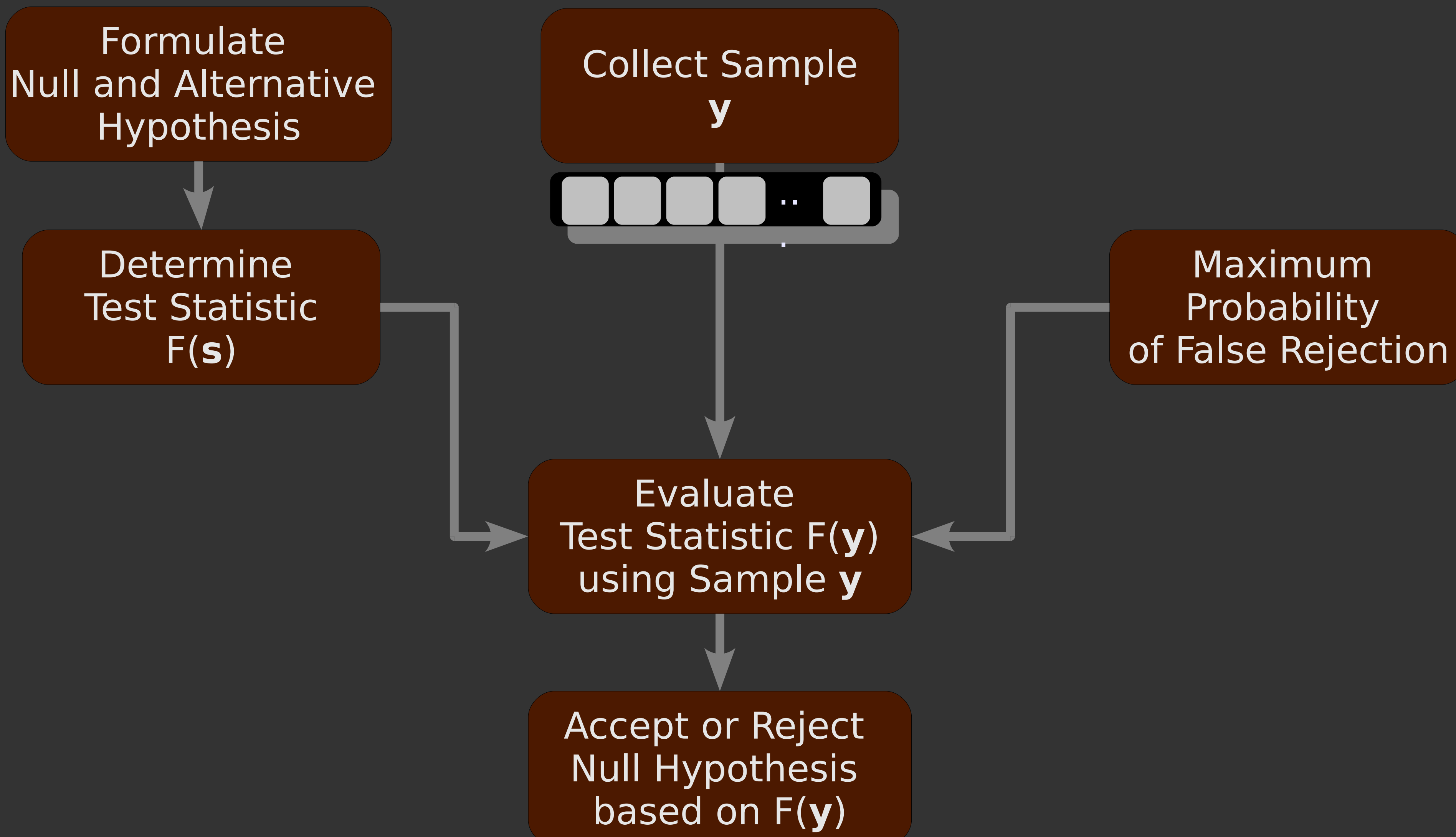
4. Estimator vs Estimator



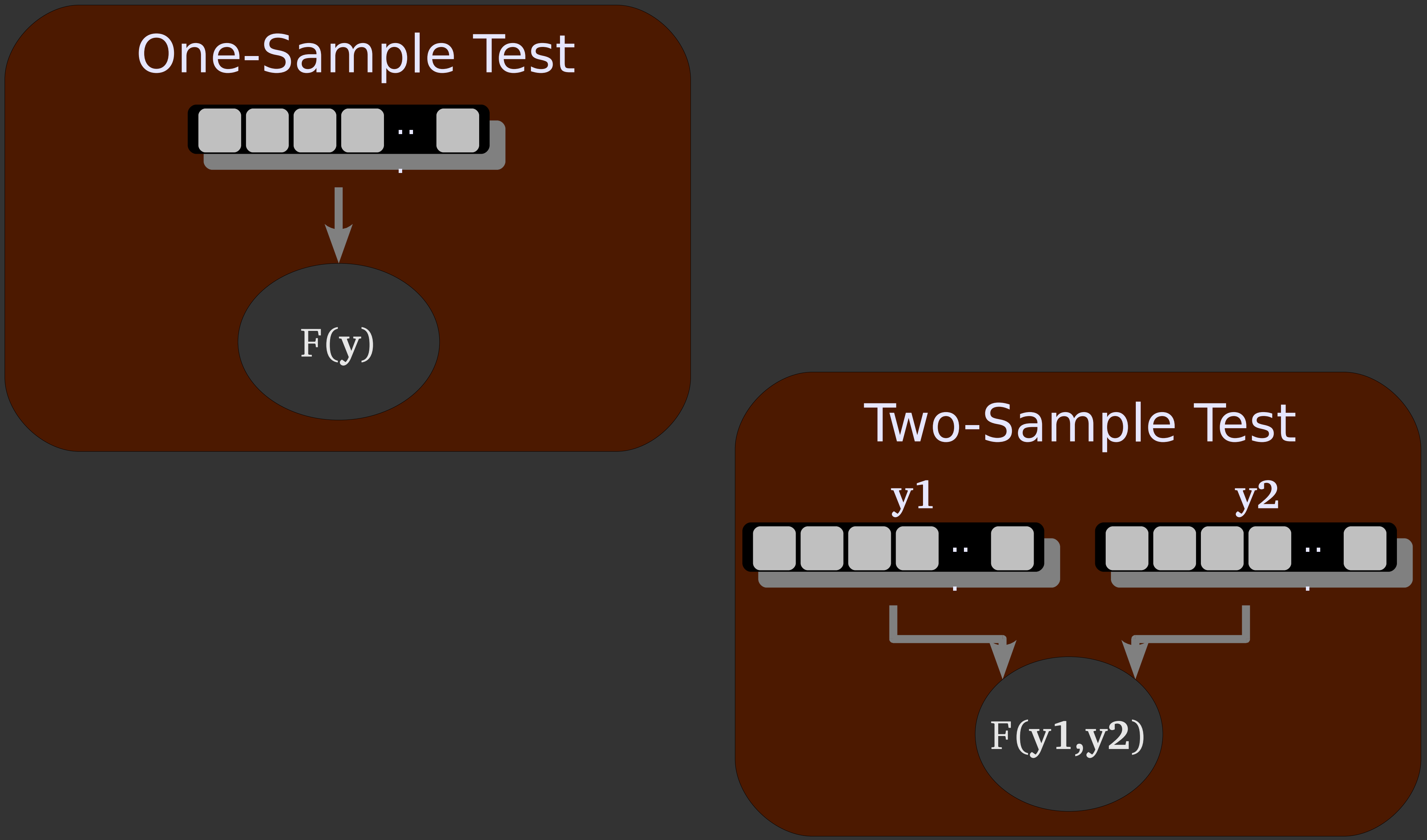
Sample: Collection of observations



Review: Hypothesis Testing

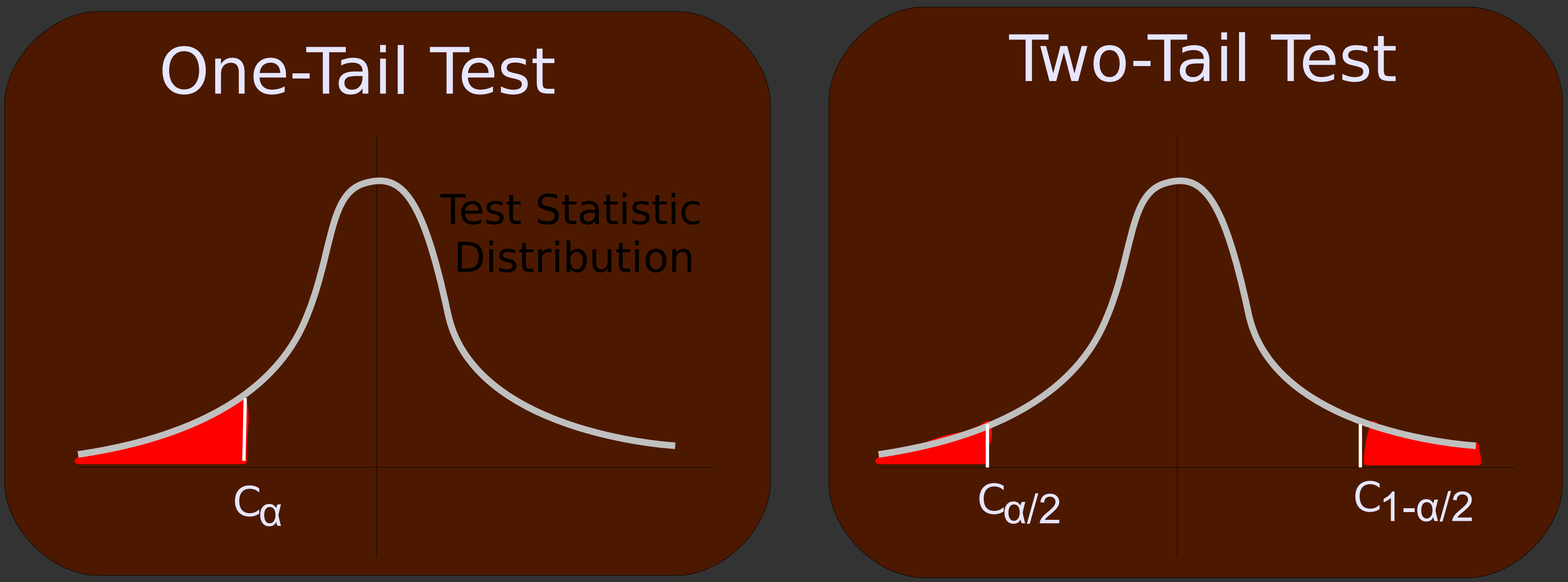


Review: One-Sample vs Two-Sample Tests



Review: Rejecting the Null-Hypothesis

- Find boundaries of rejection region
- Compute F(**y**) using the sample '**y**'
- Reject if F(**y**) falls inside rejection region



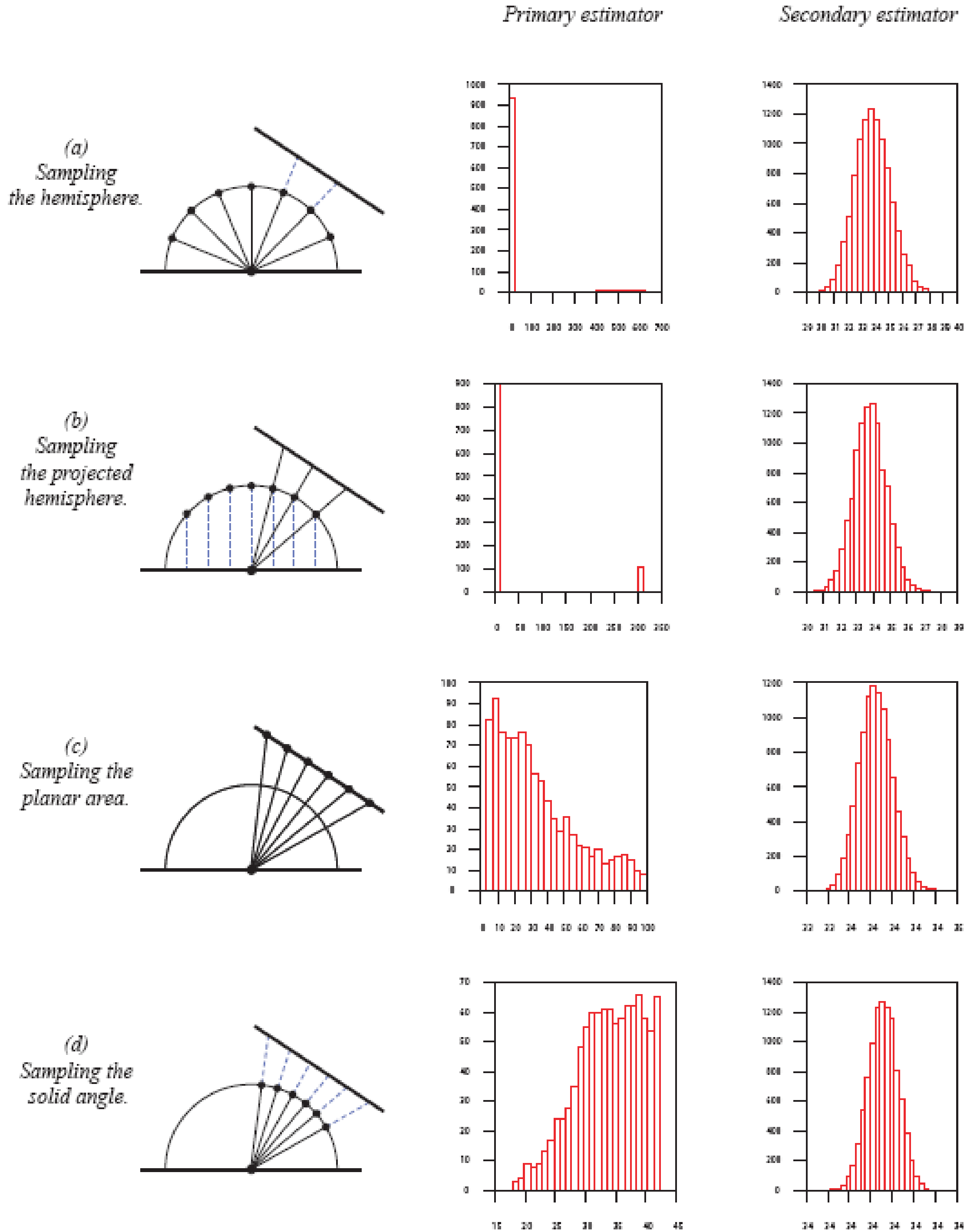
$C_\alpha = G^{-1}(\alpha)$ where G(s) is the CDF of F(s)

Tests Performed and their Test Statistics

	One-Sample Tests	**Two-Sample Tests**
Test for Mean	Test for Bias against Constant Student's t-distribution	Compare Means of Two Estimators Student's t-distribution
Test for Variance	Test that Variance is Bounded Chi-Square distribution	Compare Variances of Two Estimators F-Distribution

Setting up Hypothesis Tests

- Careful
  - Sensitive to distribution
  - most tests for normally distributed data
- Testing Estimators in Image Synthesis
  - Compare secondary instead of primary estimators


Primary estimator
Secondary estimator
(a)
Sampling
the hemisphere.
(b)
Sampling
the projected
hemisphere.
(c)
Sampling the
planar area.
(d)
Sampling the
solid angle.

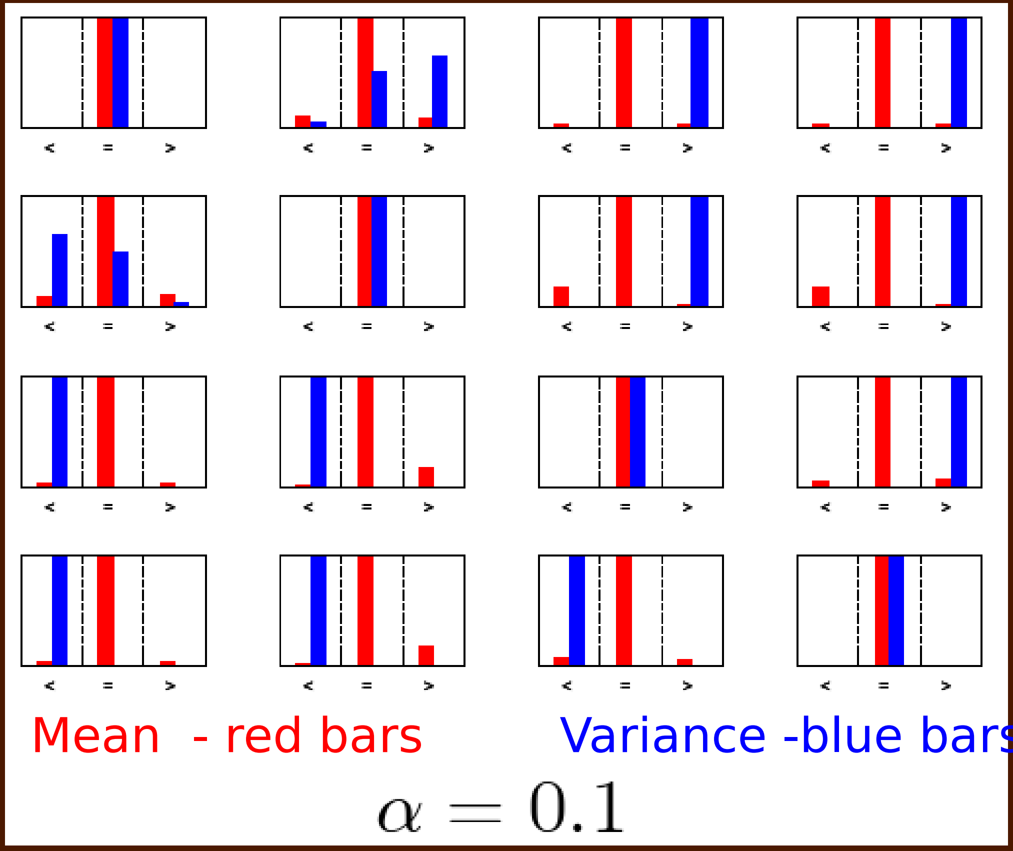
Results: Comparing Means and Variances

Uniform Hemisphere

Uniform Proj-Hemisph.

Uniform Area on Light

Uniform Solid Angle



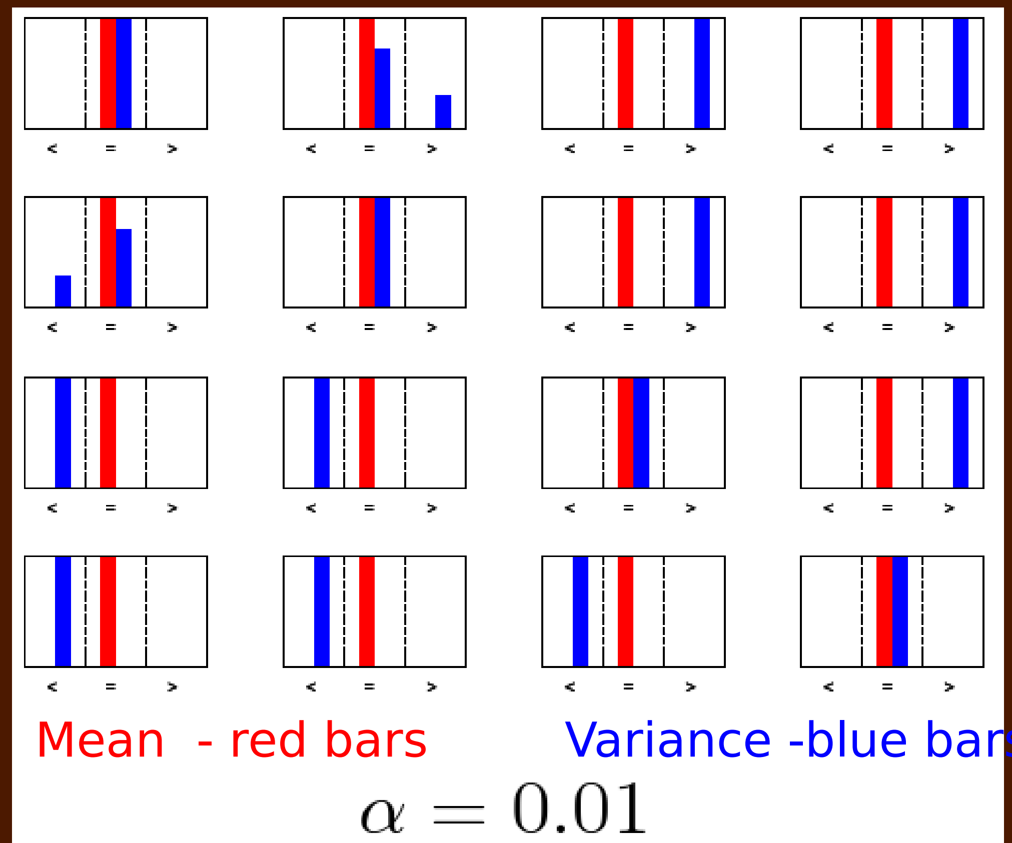
Results: Comparing Means and Variances

Uniform Hemisphere

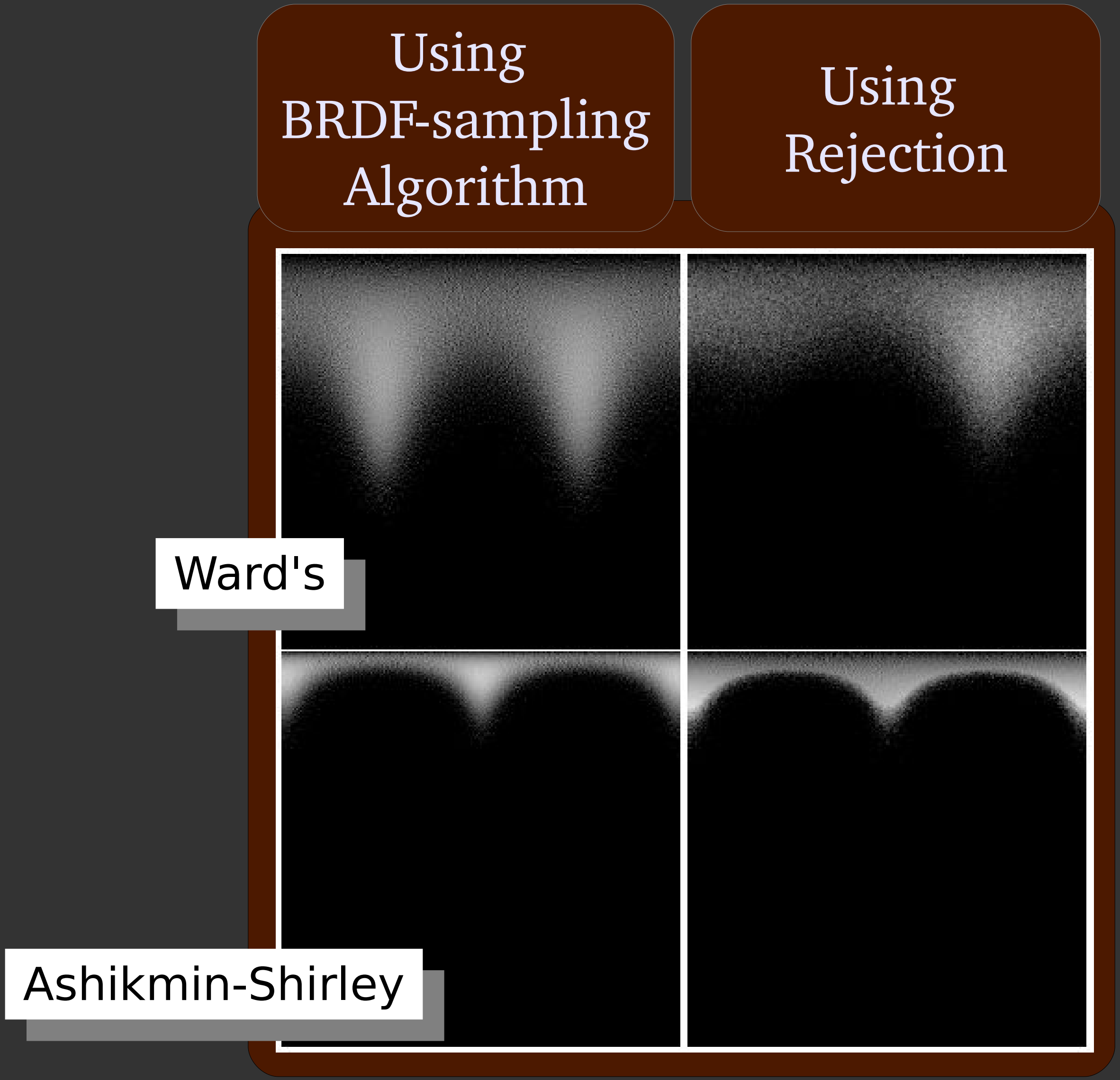
Uniform Proj-Hemisph.

Uniform Area on Light

Uniform Solid Angle

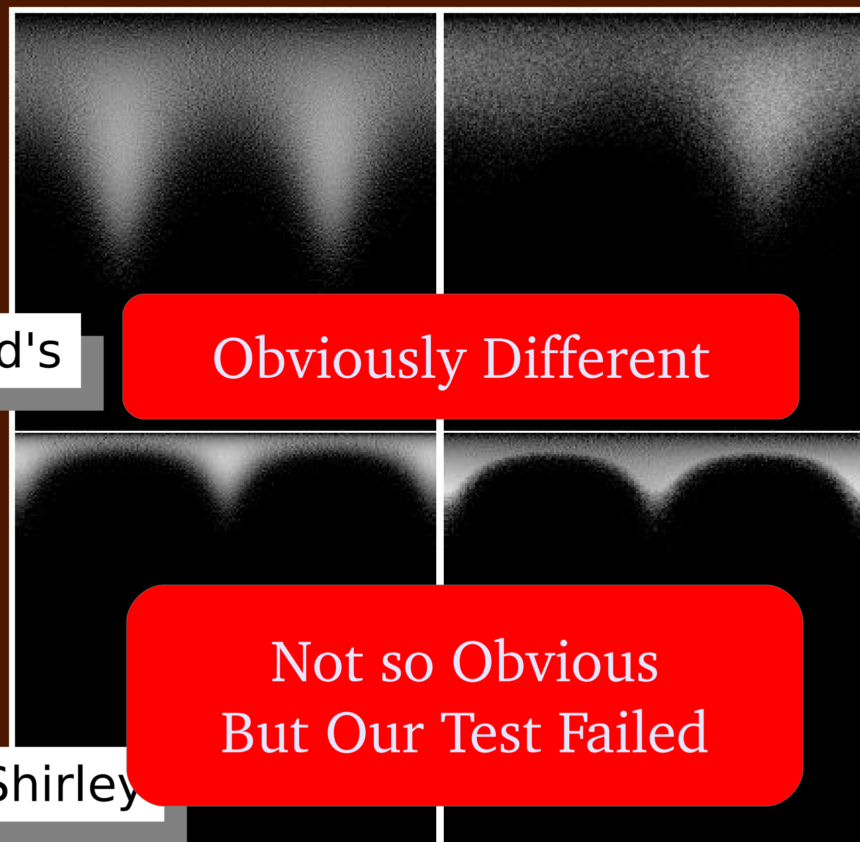


BRDF Sampling



Results – BRDF Sampling

2-Sample Goodness-of-fit (Kolmogorov-Smirnov)



Irradiance due to a Triangular Luminaire

Light Source Normal

Shading Normal

$$E(\mathbf{x}) = \int_{Area(\triangle)} L(\mathbf{x},\mathbf{z}) \frac{\mathbf{n}\cdot\mathbf{z}}{\|\mathbf{z}\|} \frac{\mathbf{n}_\triangle \cdot \mathbf{z}}{\|\mathbf{z}\|^3} \, d\mathbf{y}$$

Irradiance

Light Source

Radiance

z = x - y

Irradiance due to a Triangular Luminaire

$$E(\mathbf{x}) = \int_{Area(\triangle)} L(\mathbf{x},\mathbf{z}) \frac{\mathbf{n}\cdot\mathbf{z}}{\|\mathbf{z}\|} \frac{\mathbf{n}_\triangle \cdot \mathbf{z}}{\|\mathbf{z}\|^3}\, d\mathbf{y}$$

- Create Erroneous Estimators
  - Omitting the cosine term for shading

Irradiance due to a Triangular Luminaire

$$E(\mathbf{x}) = \int_{Area(\triangle)} L(\mathbf{x}, \mathbf{z}) \frac{\mathbf{n} \cdot \mathbf{z}}{\|\mathbf{z}\|} \frac{\mathbf{n}_{\triangle} \cdot \mathbf{z}}{\|\mathbf{z}\|^3} \, d\mathbf{y}$$

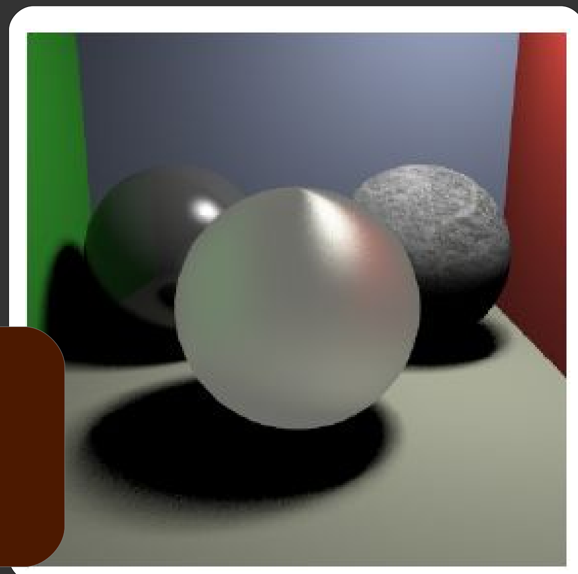
- Create Erroneous Estimators
  - Omitting the cosine term for shading
  - Non-uniform sampling of illuminaire

Irradiance due to a Triangular Luminaire

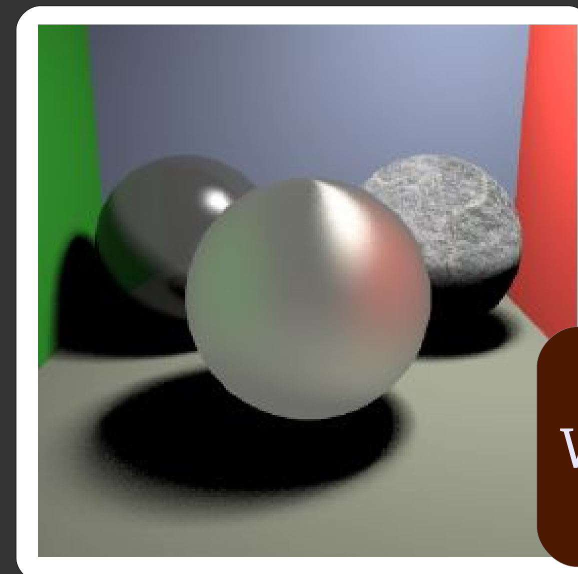
$$E(\mathbf{x}) = \int_{Area(\triangle)} L(\mathbf{x}, \mathbf{z}) \frac{\mathbf{n} \cdot \mathbf{z}}{\|\mathbf{z}\|} \frac{\mathbf{n}_{\triangle} \cdot \mathbf{z}}{\|\mathbf{z}\|^3} \, d\mathbf{y}$$

- Create Erroneous Estimators
  - Omitting the cosine term for shading
  - Non-uniform sampling of illuminaire
  - Omitting change of variables

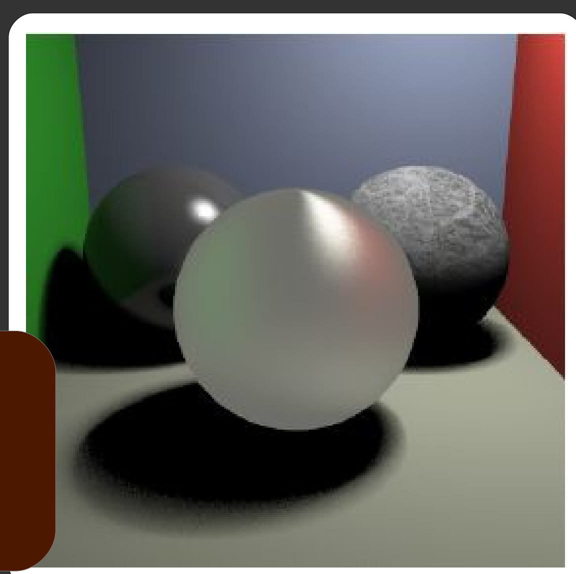
Results – Error Detection



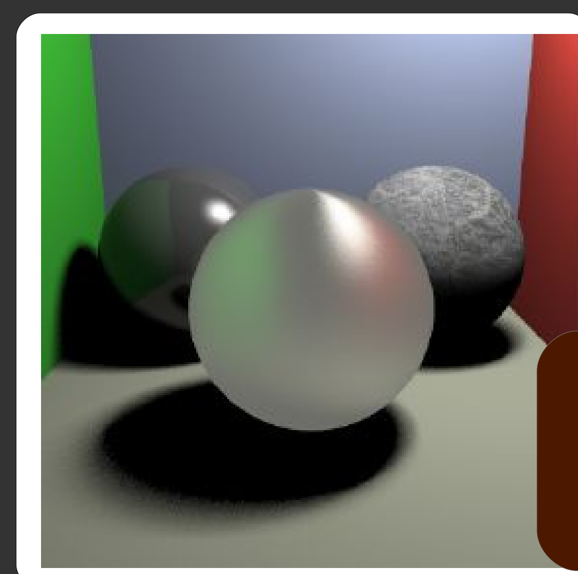
Reference



Without Cosine

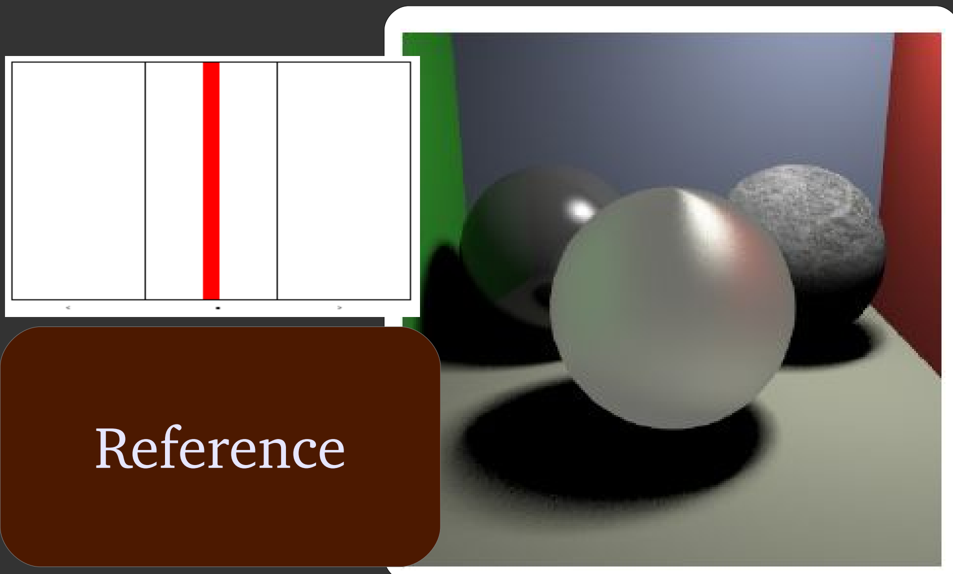


Non-uniform Sampling of Light Source

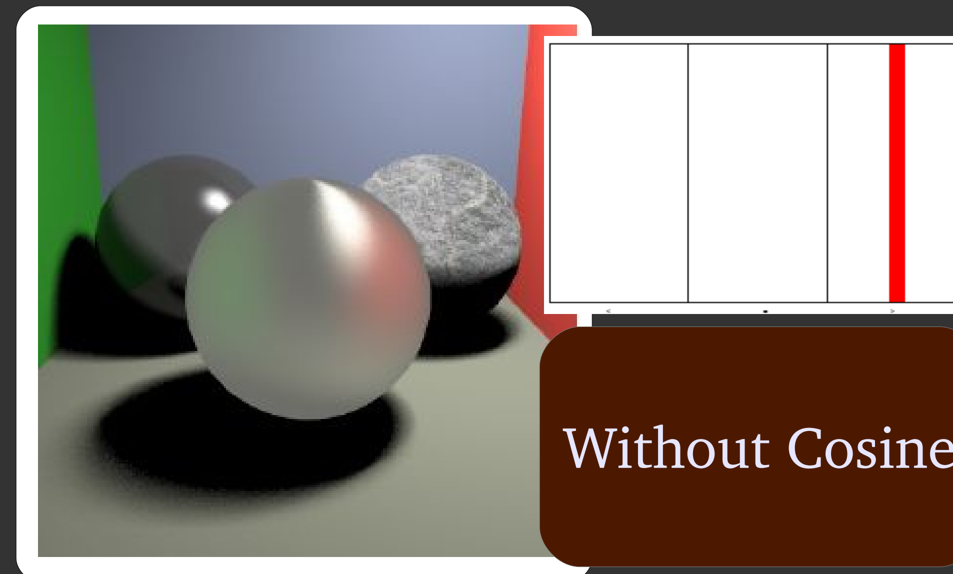


Incorrect Change of Variables

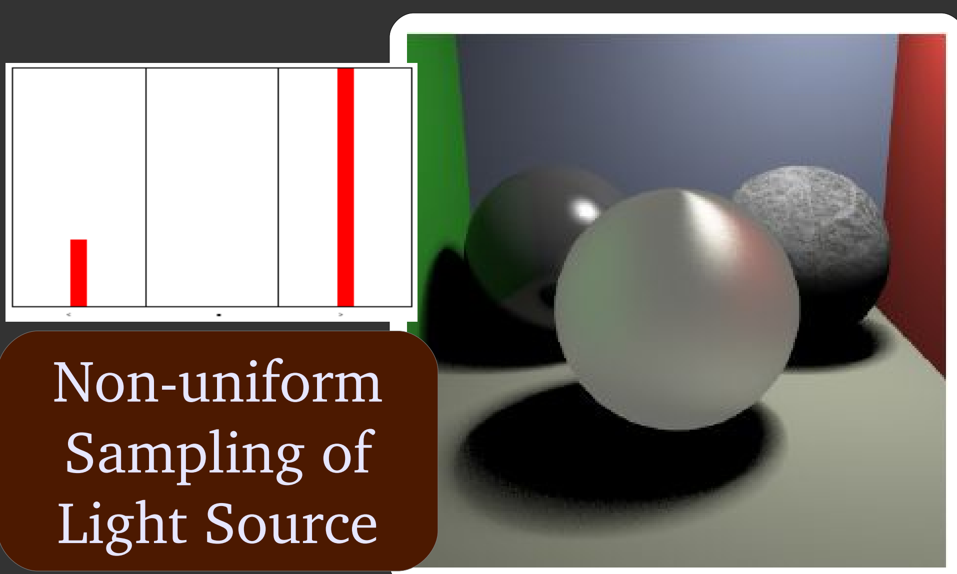
Results – Error Detection



Reference



Without Cosine



Non-uniform Sampling of Light Source

Incorrect Change of Variables

Questions ?

[Fisher 59]

Statistical Methods and Scientific Inference

[Neyman & Pearson 28]

On the Use and Interpretation of Certain Test Criteria for Purposes of Statistical Inference

[Freund & Walpole 87]

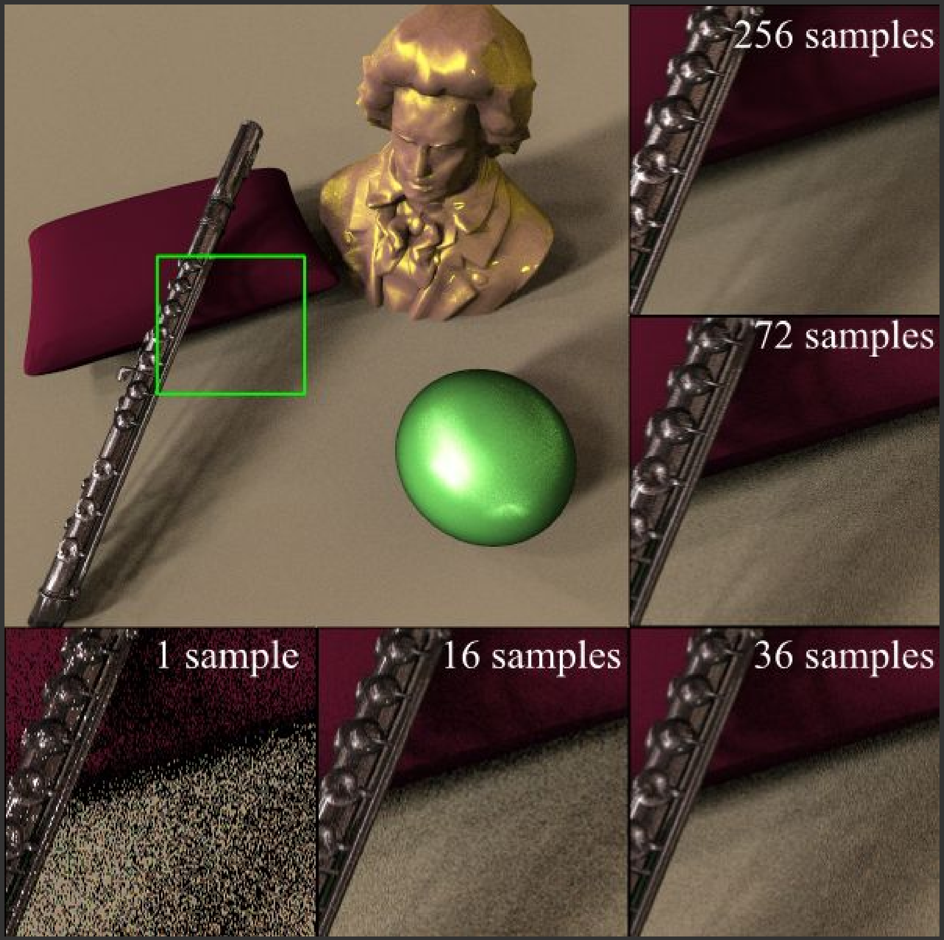
Mathematical Statistics

Contributions – 1

- Bandwidth prediction for efficient depth of field

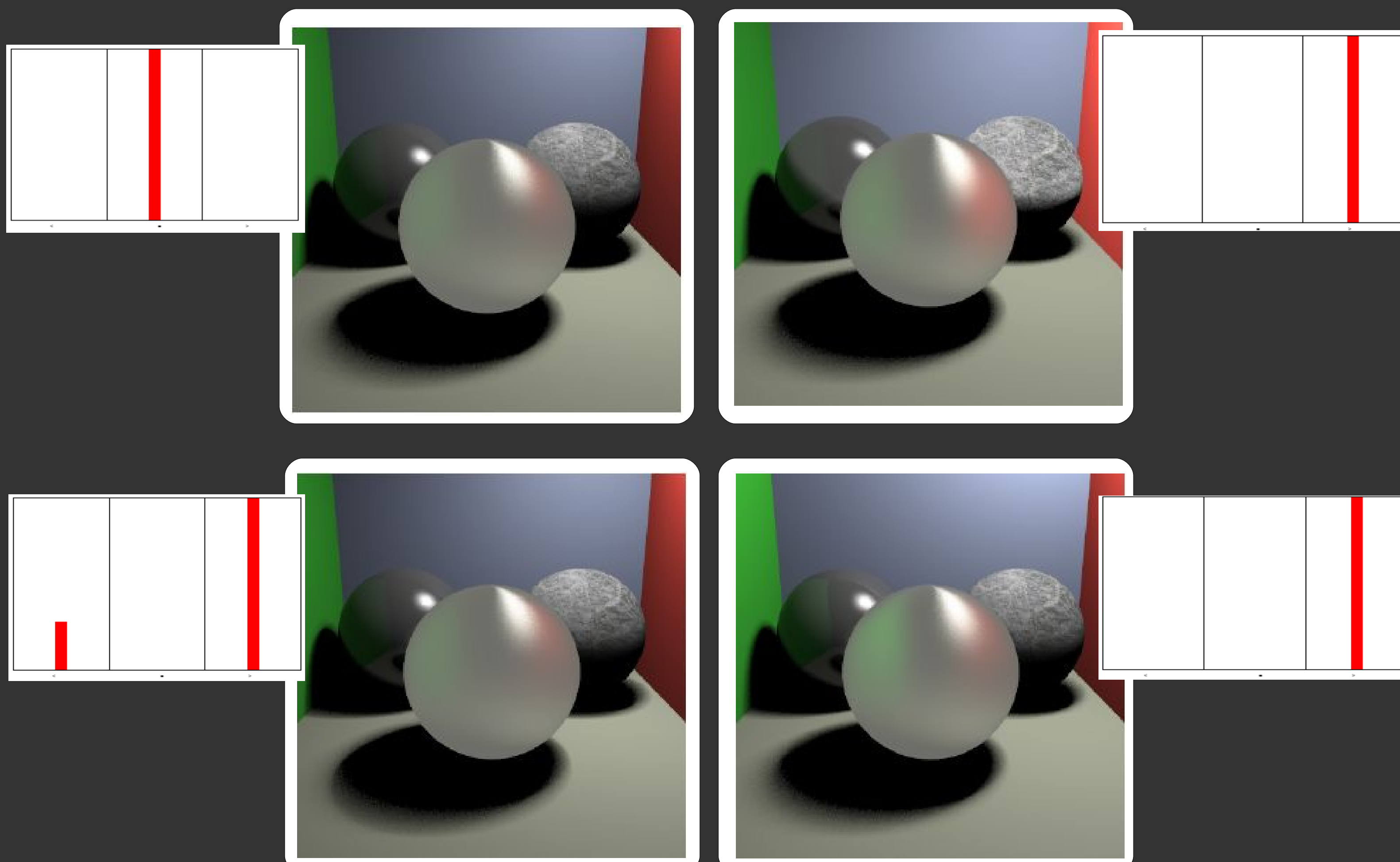
Contributions – 2

- Steerable importance sampling



Contributions – 3

- Framework to assess Monte Carlo estimators

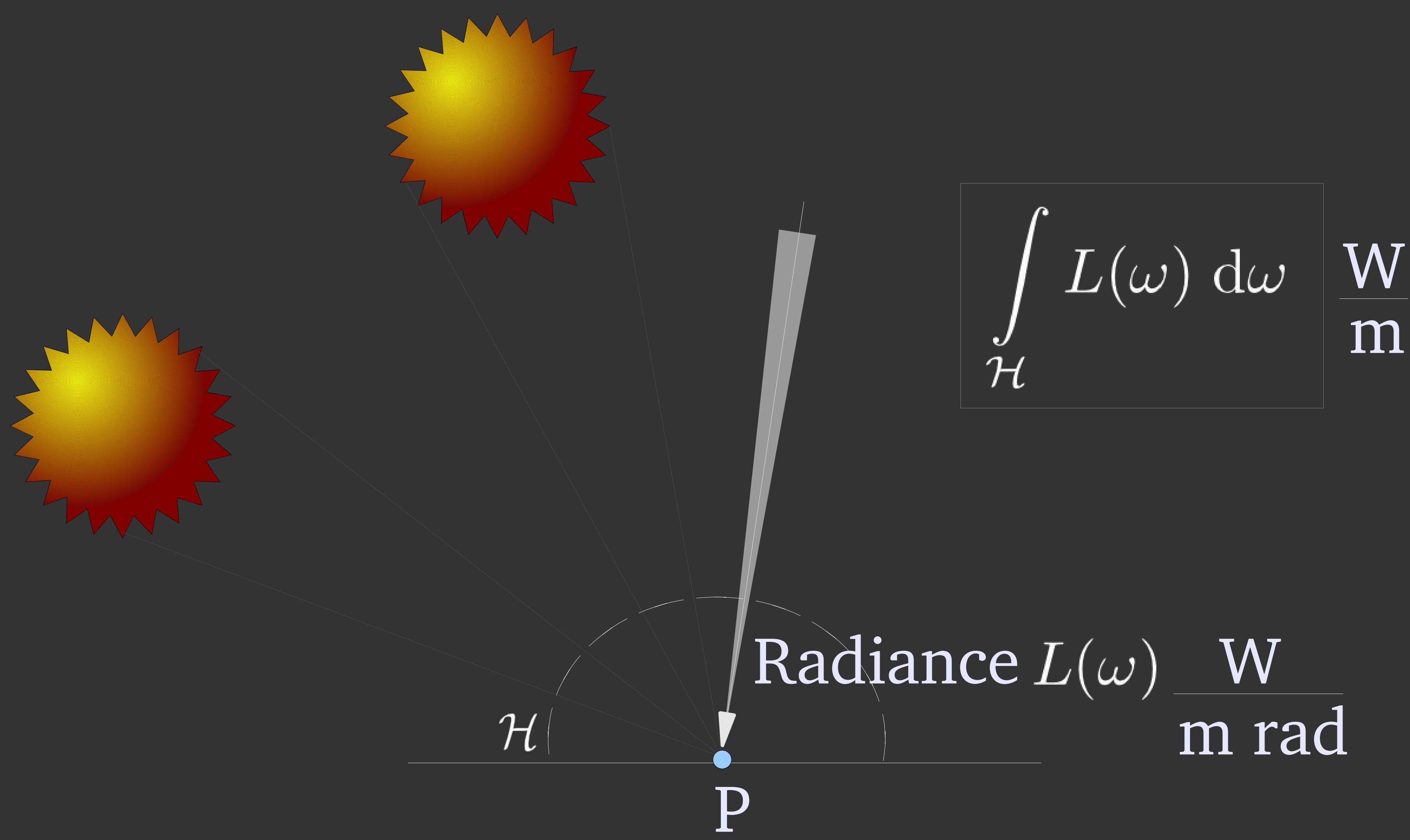


Questions ?

Acknowledgements

- Kitchen scene images – Cyril Soler
- INRIA, Grenoble
- James Arvo, Fredo Durand, Nicolas Holzschuch, Francois Sillion

Incident Illumination at P



Monte Carlo integration

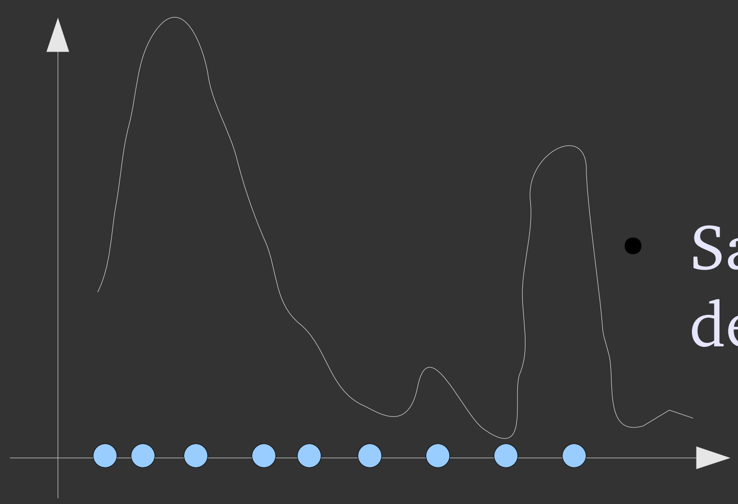
$$\int_{\mathcal{H}} L(\omega)\,\mathrm{d}\omega \approx \frac{1}{N}\sum_{i=1}^{N} L(\omega_i)$$

- ω_i are distributed uniformly in $\mathcal{H}$

Monte Carlo integration

$$\int_{\mathcal{H}} L(\omega)\,\mathrm{d}\omega \approx \frac{1}{N}\sum_{i=1}^{N} L(\omega_i)$$

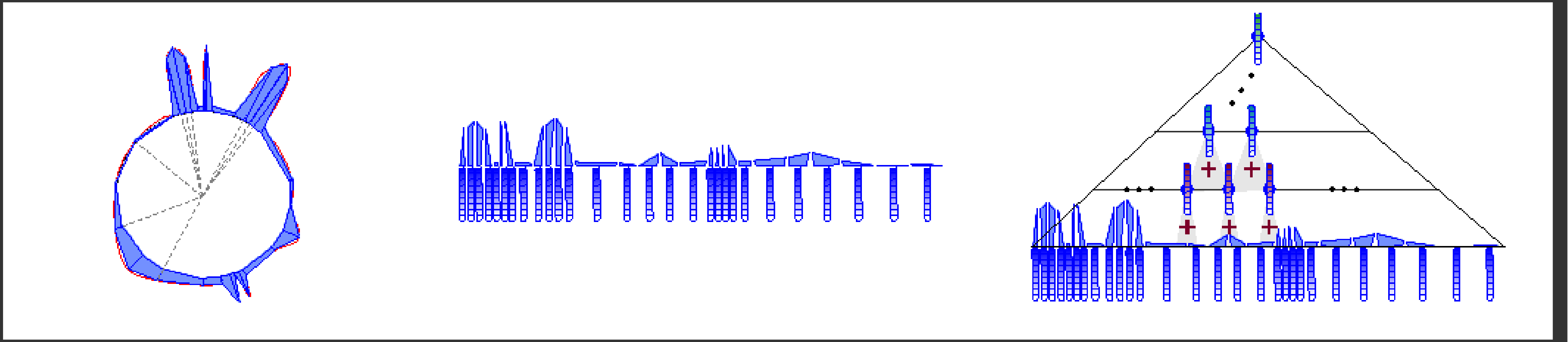
- ω_i are distributed uniformly in $\mathcal{H}$



- Sample distribution determines efficiency

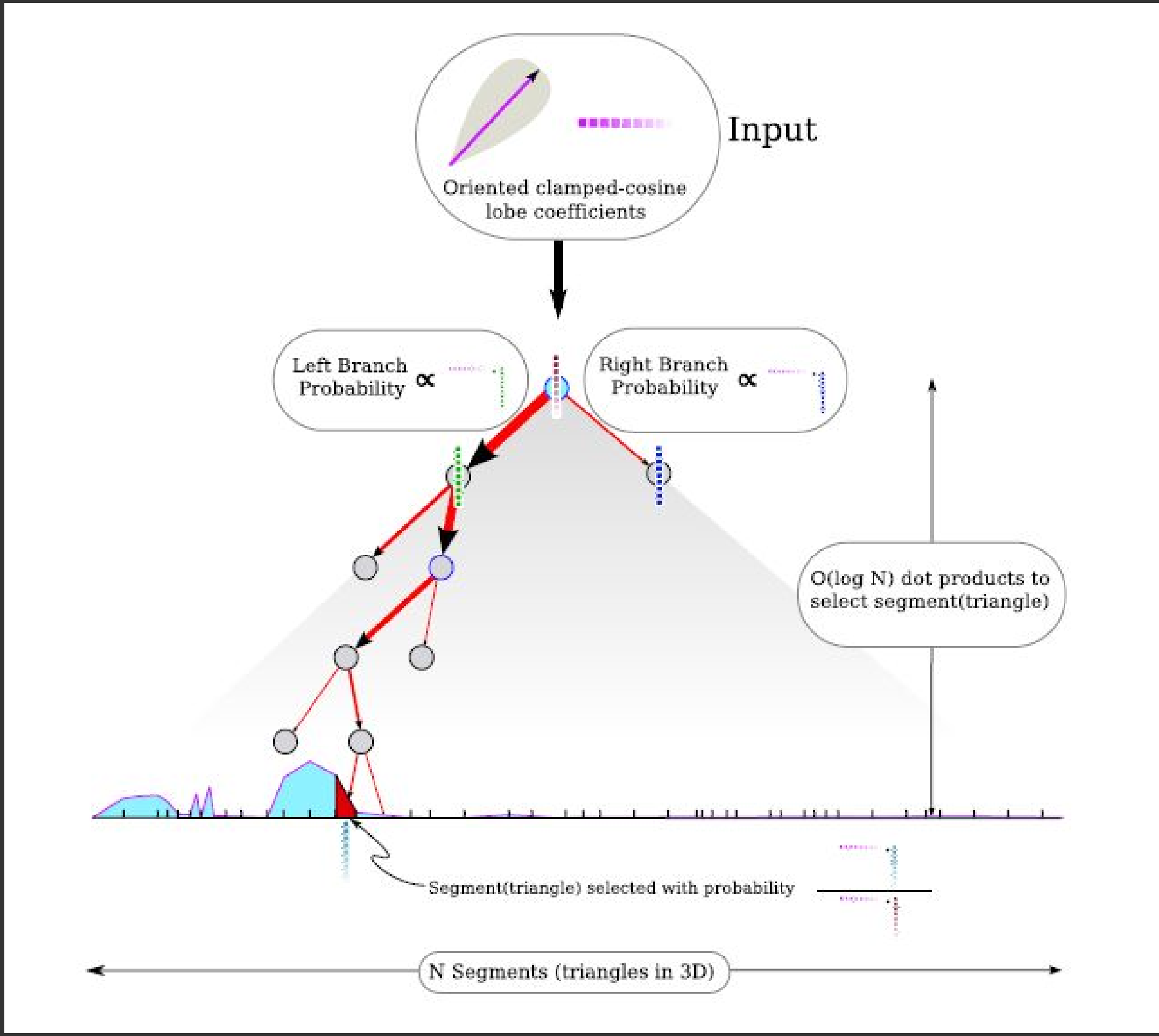
Preprocess

- Partition sphere of directions into a set of spherical triangles
  - Compute and store vector **w** at each vertex
  - Compute and store vector **W** for each triangle
- Construct hierarchy of weights(no higher order terms)



Sample Generation

- Tree traversal to select a triangle given a normal **n**



Frequency Transport