Steerable Importance Sampling for Efficient Direct Distant Illumination

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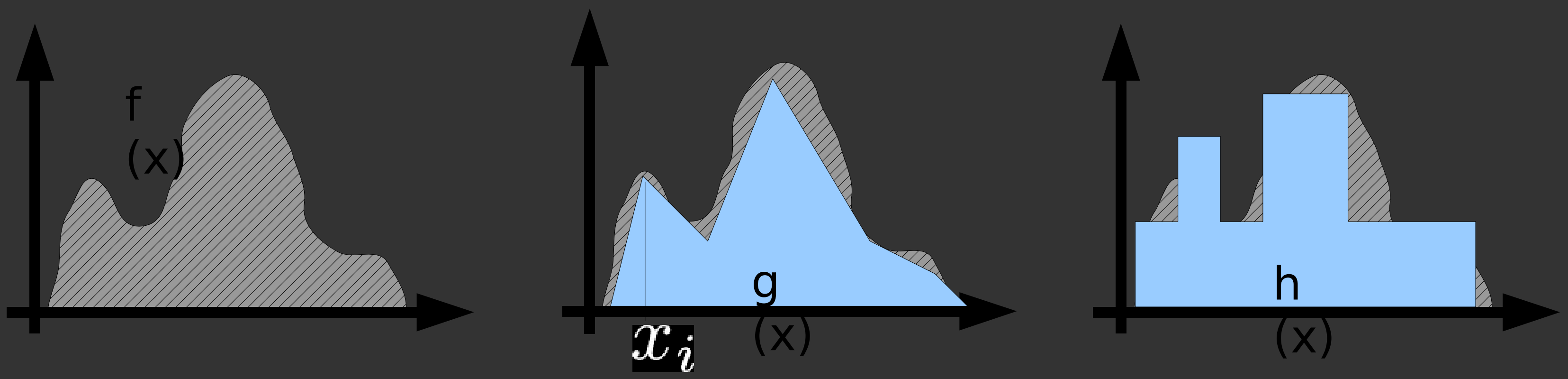
Review: Importance sampling

$$\int_{\mathcal{D}} f(x)\,\mathrm{d}x \;\approx\; \frac{1}{N}\int_{\mathcal{D}} g(x)\,\mathrm{d}x\;\sum_{i=1}^{N}\frac{f(x_i)}{g(x_i)}$$

where $x_i \;\sim\; g(x)$

Review: Importance sampling

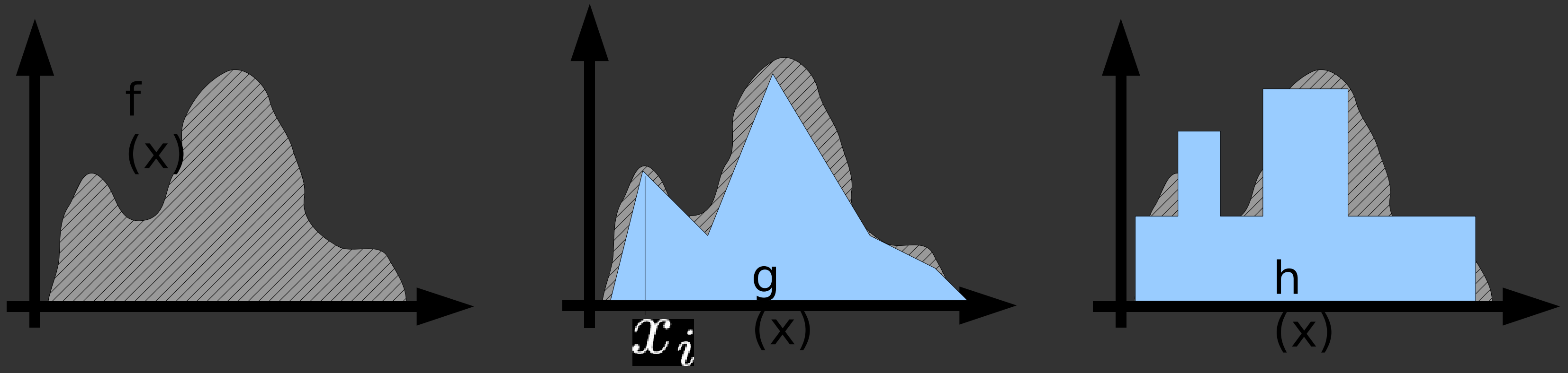
$$\int_{\mathcal{D}} f(x)\,\mathrm{d}x \;\approx\; \frac{1}{N}\int_{\mathcal{D}} g(x)\,\mathrm{d}x \sum_{i=1}^{N} \frac{f(x_i)}{g(x_i)}$$



Which is a better importance function, *g* or *h* ?

Review: Importance sampling

$$\int_{\mathcal{D}} f(x)\,\mathrm{d}x \approx \frac{1}{N}\int_{\mathcal{D}} g(x)\,\mathrm{d}x \sum_{i=1}^{N} \frac{f(x_i)}{g(x_i)}$$



Which is a better importance function, g or h ?

What if *f(x)* changes ?

Review: Steerable functions

- Transformed functions = linear combination of basis

Transformed Function

Inner Product

$$g_T(x) = < \boldsymbol{s}(T), \boldsymbol{b}(x) >$$

bases

Transformation-dependent coefficients

Direct, distant illumination

Reflected radiance along direction ω_o

$$\int_{S^2} \underbrace{V(x,\omega_i)}_{\text{Visibility}} \underbrace{\rho(\omega_o,\omega_i)}_{\text{Reflectance Function}} \underbrace{L(\omega_i)}_{\text{Incident Radiance}} \underbrace{\max(\omega_i.\mathbf{n},\ 0)}_{\text{Clamped Cosine}} \, \mathrm{d}\omega_i$$

Direct, distant illumination

Reflected radiance along direction ω_o

$$\int_{S^2} V(x,\omega_i)\rho(\omega_o,\omega_i)L(\omega_i)\max(\omega_i.\mathbf{n},\,0)\,\mathrm{d}\omega_i$$

Visibility

Reflectance Function

Incident Radiance x Clamped Cosine

Importance Function

Domain Partitioning

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n})\; d\omega_i$$

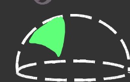
Partition into Spherical Triangles

$$\left(\int + \int + \int + \ldots\right) \gamma(x, \omega_i, \omega_r, \mathbf{n})\; d\omega_i$$

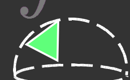
Change of variables – 1

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n})\, d\omega_i$$



Spherical to planar triangle

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n})\, \varphi(\omega_i, \mathbf{n}_\triangle)\, d\omega_i$$



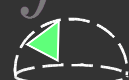
Planar triangle normal

Change of variables – 2

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n})\, d\omega_i$$

Spherical to planar triangle

$$\int \gamma(x, \omega_i, \omega_r, \mathbf{n})\, \varphi(\omega_i, \mathbf{n}_\triangle)\, d\omega_i$$



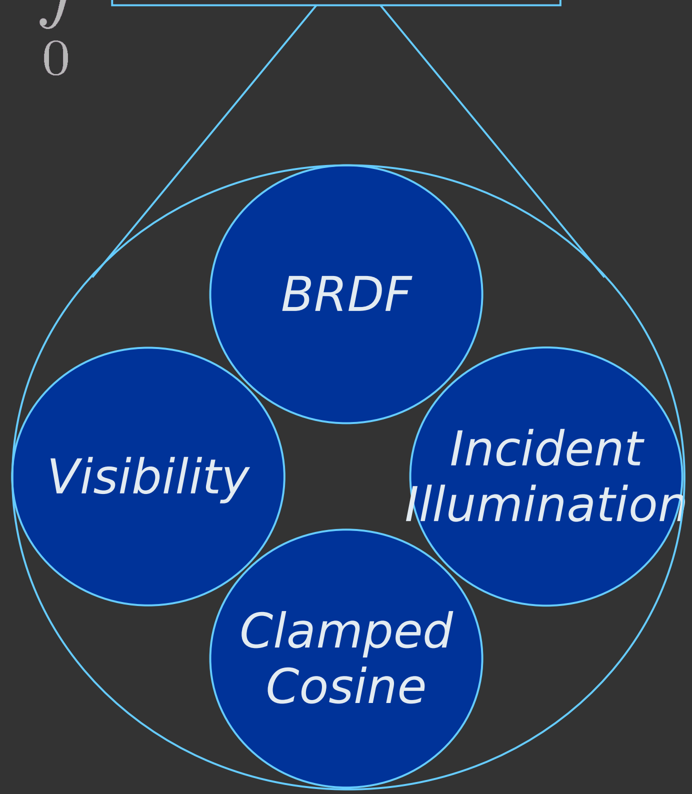
Unit square to triangle parameterization

$$\int_0^1 \int_0^1 \gamma(x, \omega_i, \omega_r, \mathbf{n})\, \varphi(p_0, p_1, \mathbf{n}_\triangle)\, \left|J(p_0, p_1)\right|\, dp_0\, dp_1$$

Jacobian of Parameterization

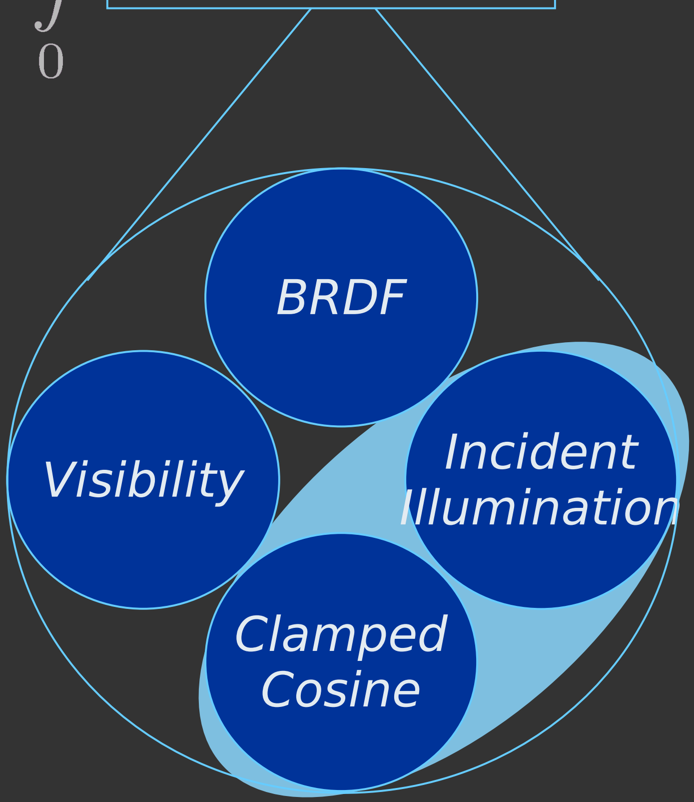
Novel parameterization

$$\int_0^1 \int_0^1 \boxed{\gamma(x, \omega_i, \omega_r, \mathbf{n})}\, \varphi(p_0, p_1, \mathbf{n}_\triangle)\, |J(p_0, p_1)|\, dp_0\, dp_1$$



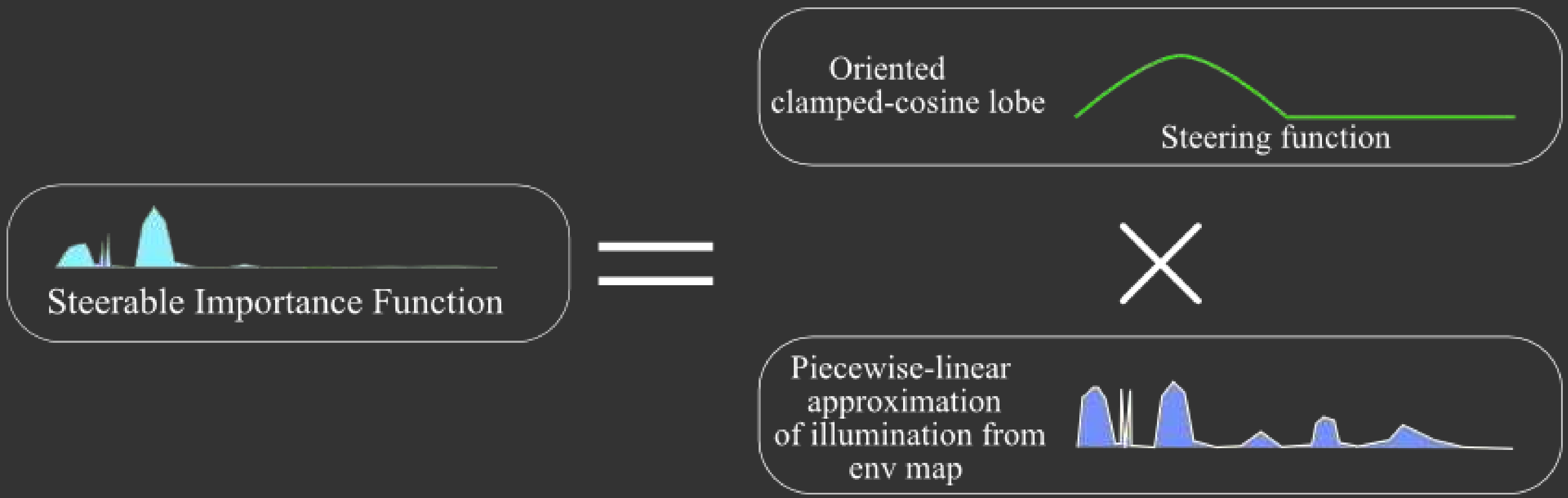
Novel parameterization

$$\int_0^1 \int_0^1 \gamma(x, \omega_i, \omega_r, \mathbf{n}) \, \varphi(p_0, p_1, \mathbf{n}_\triangle) \, |J(p_0, p_1)| \, dp_0 \, dp_1$$

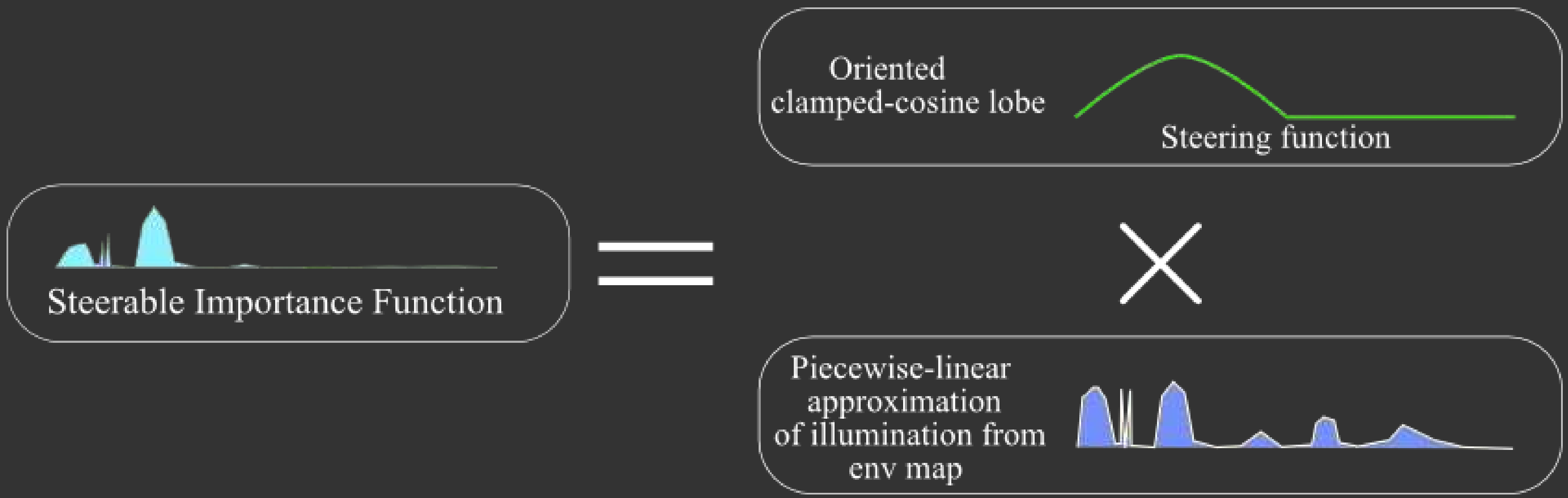


Derive parameterization so that
Jacobian $\approx$ Illumination * Clamped Cosine

Steerable importance function



Steerable importance function



Is this steerable?

Steerable importance function

$$g_{\mathbf{n}}(\mathbf{u}) = L(\mathbf{u}) \max(\mathbf{n}.\mathbf{u}, 0)$$

Steerable importance function

$$g_{\mathbf{n}}(\mathbf{u}) = L(\mathbf{u})\ \max(\mathbf{n}.\mathbf{u}, 0)$$

Represent using SH bases
8 coefficients – good approximation
[Ramamoorthi & Hanrahan]

$$<\mathbf{a}(\mathbf{n}), \mathbf{Y}(\mathbf{u})>$$

Rotated coefficients

Spherical Harmonic bases

Steerable importance function

$$g_n(\mathbf{u}) = L(\mathbf{u})\ \max(\mathbf{n}.\mathbf{u}, 0)$$

$$\langle \mathbf{a}(\mathbf{n}), \mathbf{Y}(\mathbf{u}) \rangle$$

rewrite

$$g_n(\mathbf{u}) = \langle \mathbf{a}(\mathbf{n}), L(\mathbf{u})\ \mathbf{Y}(\mathbf{u}) \rangle$$

Steerable importance function

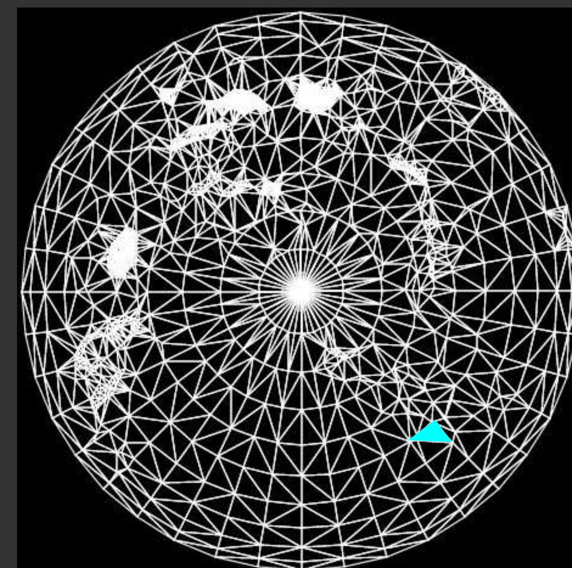
$$g_n(\mathbf{u}) = L(\mathbf{u}) \max(\mathbf{n.u}, 0)$$

$$\langle \mathbf{a(n)}, \mathbf{Y(u)} \rangle$$

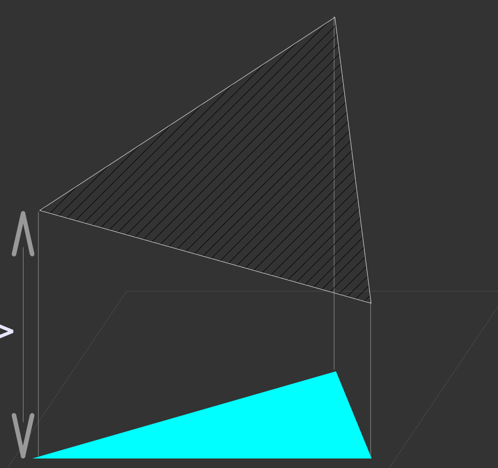
$$g_n(\mathbf{u}) = \langle \mathbf{a}(\mathbf{n}), L(\mathbf{u})\, \mathbf{Y(u)} \rangle$$

Precomputed

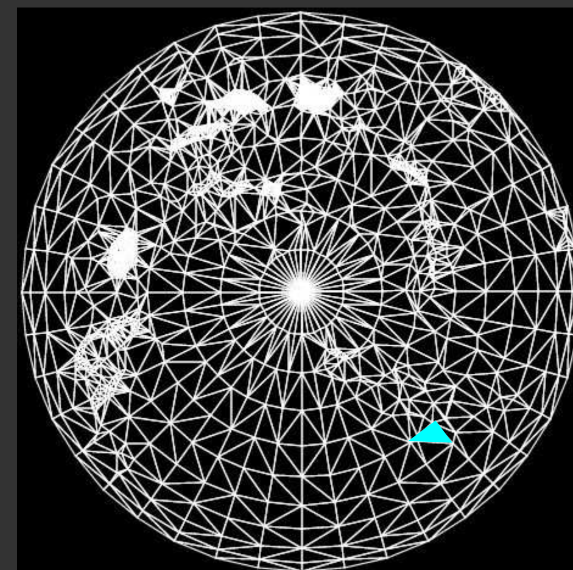
Steerable importance function



$\langle \mathbf{a}(\mathbf{n}), \mathrm{L}(\mathbf{u})\ \mathbf{Y}(\mathbf{u}) \rangle$



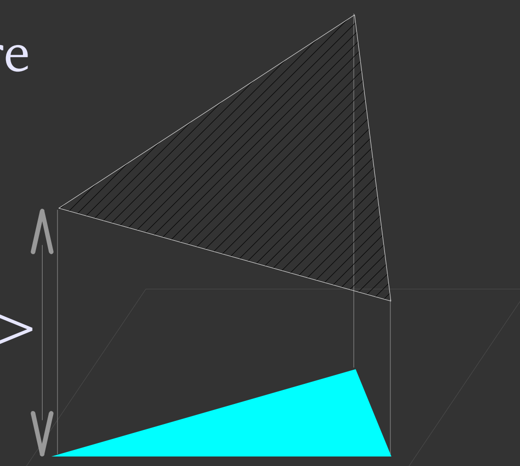
Steerable importance function



Precompute and store
(per vertex)

Function of normal

$\langle \mathbf{a}(\mathbf{n}), \mathrm{L}(\mathbf{u})\ \mathbf{Y}(\mathbf{u}) \rangle$



Drawing samples

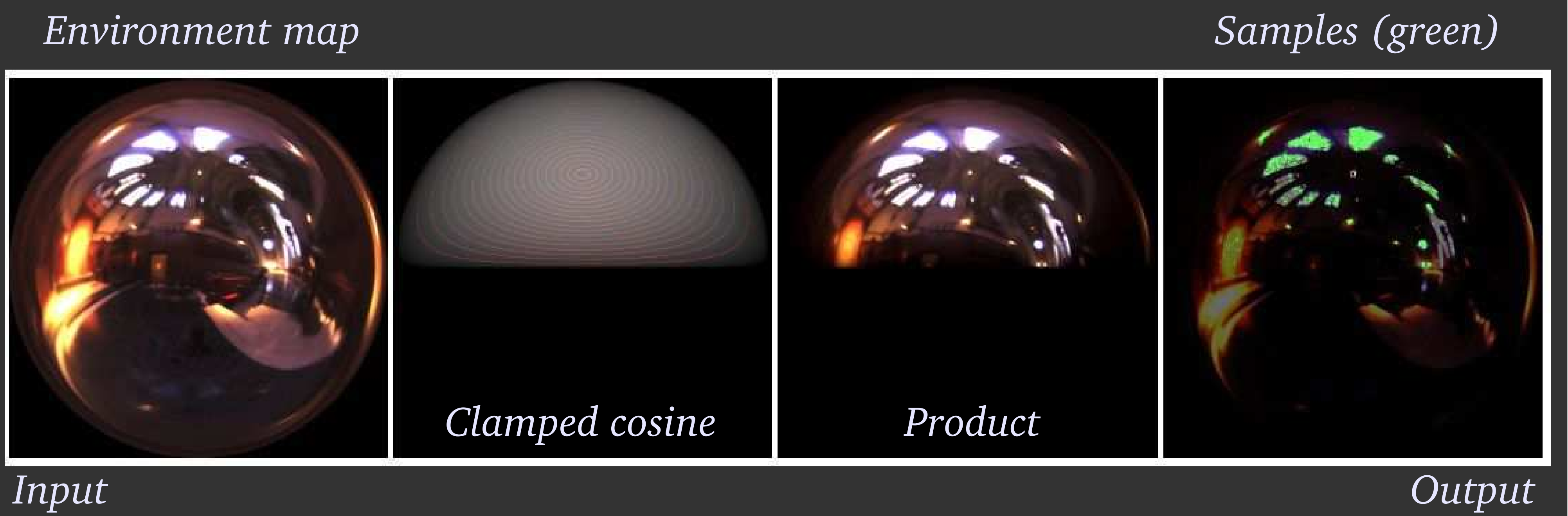
- Triangle selection
- Stratified sampling of selected triangle

Drawing samples

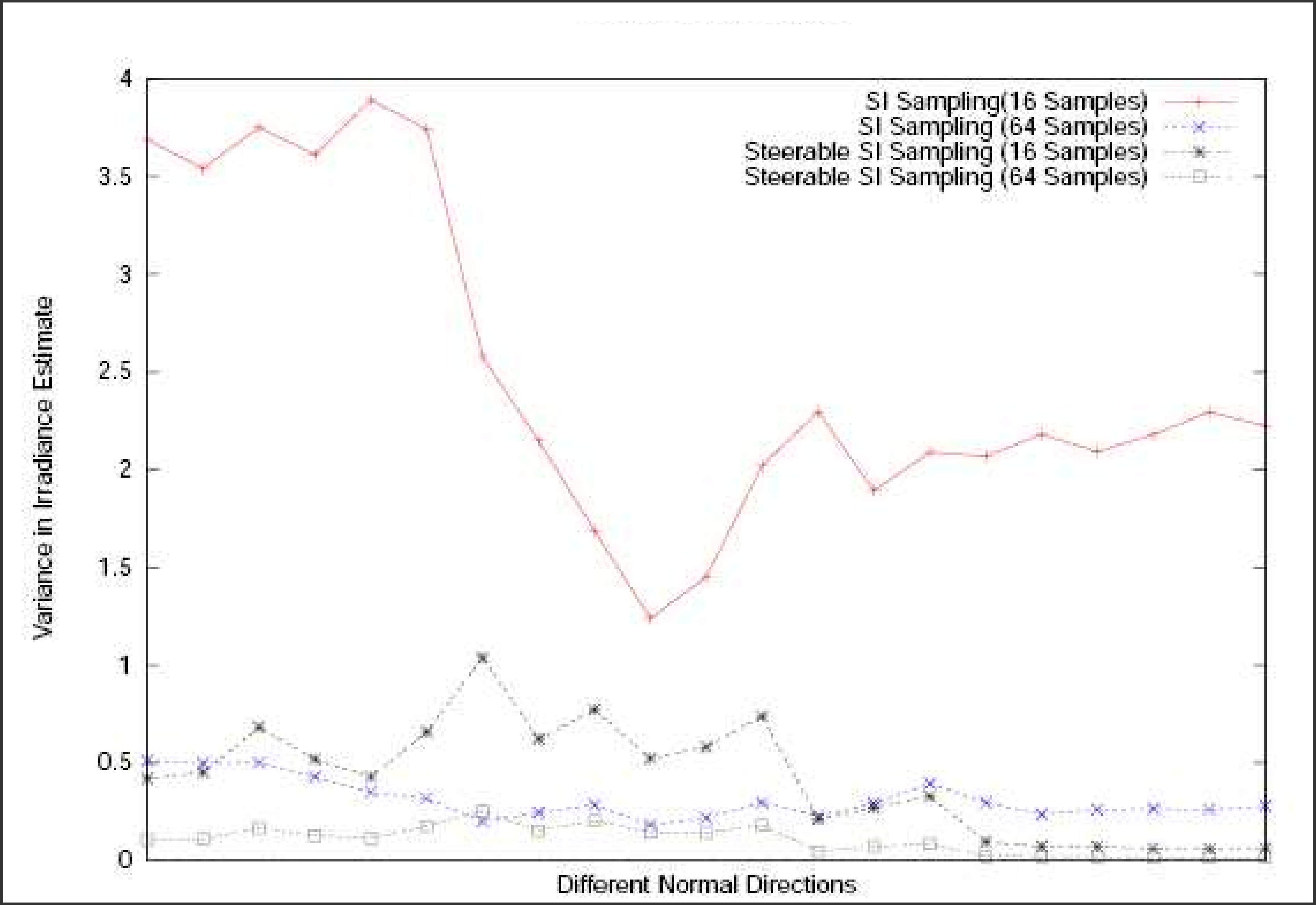
- Triangle selection
  - proportional to function integral within triangle
  - O(log *N*) cost (N triangles)

- Stratified sampling of selected triangle
  - according to linear function
  - O(1) cost

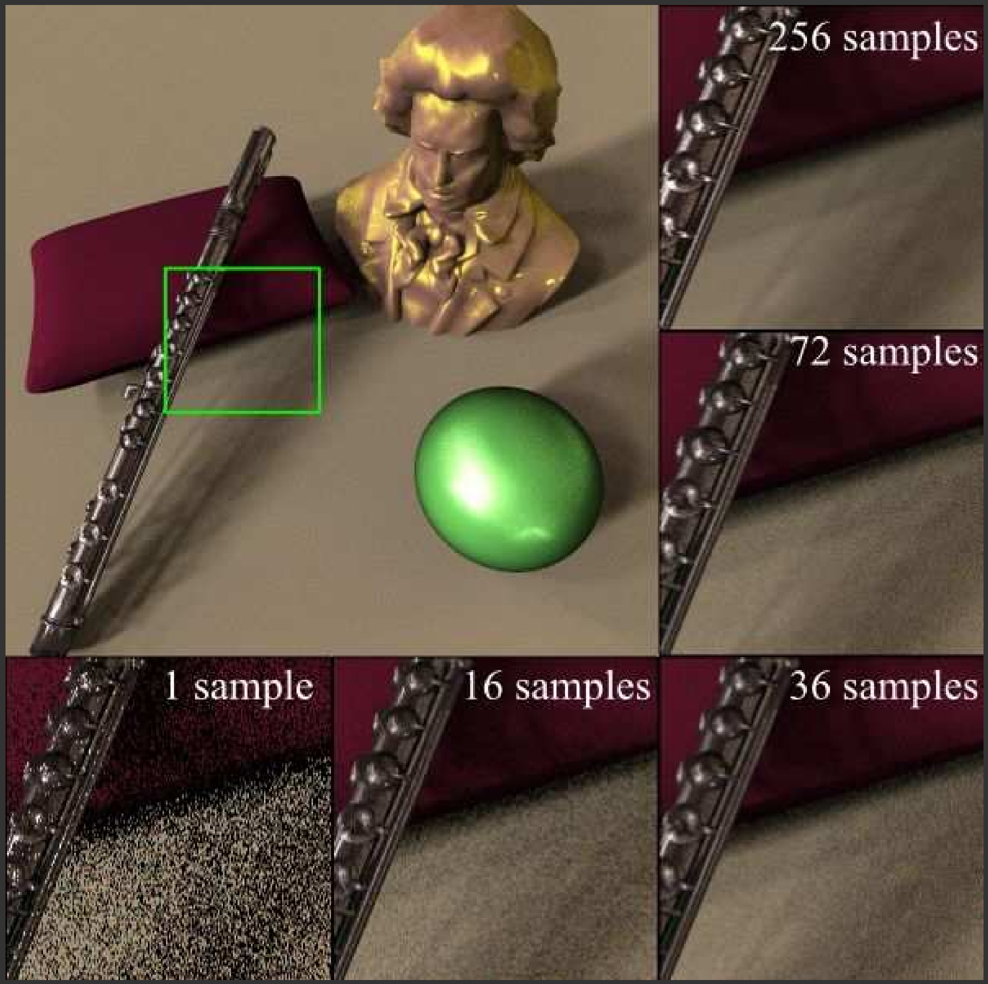
Results

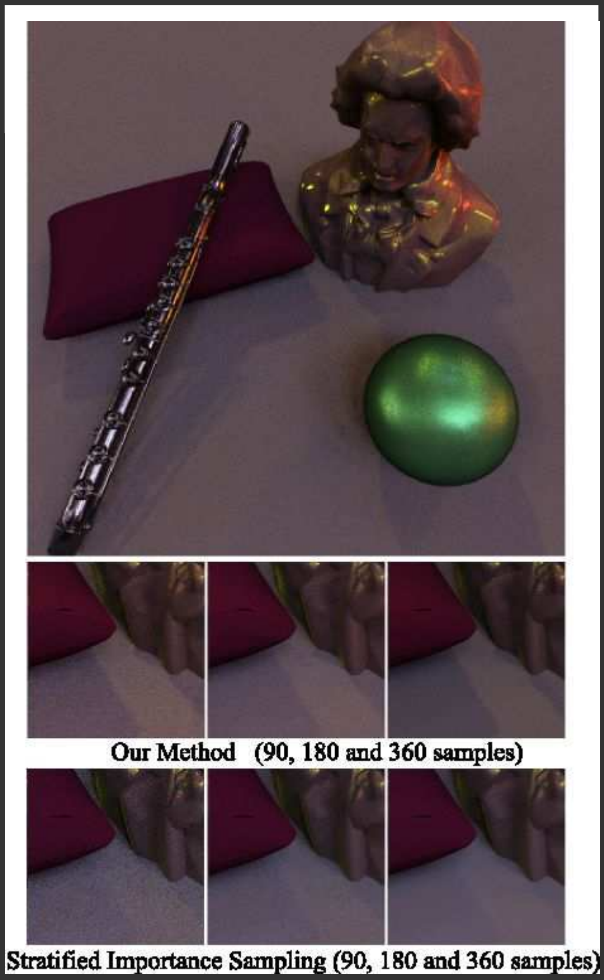

Environment map
Samples (green)
Clamped cosine
Product
Input
Output

Results: Reduced variance



Results: Images generated





Questions ?

[Ramamoorthi & Hanrahan]

An efficient representation for irradiance environment maps.

SIGGRAPH 2001.

[Teo]

Theory and applications of steerable functions.

PhD thesis, 1998.

[W. Freeman]

Steerable filters and the local analysis of image structure.

PhD thesis, 1992.